Accelerated math. Homework 9.

Problems marked with * are more difficult.

1. Write the following expressions in a shorter way substituting product with power:
   Examples: $(-a)\cdot(-a)\cdot(-a)\cdot(-a)=(-a)^4$, $3m\cdot m\cdot m\cdot 2k\cdot k\cdot k\cdot k=6m^3k^4$

a. $(-y)\cdot(-y)\cdot(-y)\cdot(-y)$;

b. $-y\cdot y\cdot y\cdot y$;

c. $(ab)\cdot(ab)\cdot(ab)\cdot(ab)\cdot(ab)\cdot(ab)$;

d. $a\cdot b\cdot b\cdot b\cdot b\cdot b$;

e. $(-5m)(-5m)\cdot 2n\cdot 2n\cdot 2n$;

f. $-5m\cdot m\cdot 2n\cdot n\cdot n$;

g. $p-q\cdot q\cdot q\cdot q\cdot q$;

h. $(p-q)\cdot(p-q)\cdot(p-q)\cdot(p-q)$;

a. $(-y)\cdot(-y)\cdot(-y)\cdot(-y)=(-y)^4$
b. $-y\cdot y\cdot y\cdot y=-y^4$
c. $(ab)\cdot(ab)\cdot(ab)\cdot(ab)\cdot(ab)\cdot(ab)=(ab)^6$
d. $a\cdot b\cdot b\cdot b\cdot b\cdot b=a\cdot b^5=ab^5$
e. $(-5m)(-5m)\cdot 2n\cdot 2n\cdot 2n=(-5m)^2(2n)^3=(-5)^2m^2 2^3n^3=25\cdot 8\cdot m^2n^3=200m^2n^3$
f. $-5m\cdot m\cdot 2n\cdot n\cdot n=-5\cdot m^2\cdot 2\cdot n^3=-10m^2n^3$
g. $p-q\cdot q\cdot q\cdot q\cdot q=p-q^5$
h. $(p-q)\cdot(p-q)\cdot(p-q)\cdot(p-q)=(p-q)^4$

2. Write the following expressions substituting power with a product of several factors:
   Examples: $(-x)^3=(-x)\cdot(-x)\cdot(-x)$; $3y-a^4=3y-a\cdot a\cdot a\cdot a$

   a. $(-n)^3$;
   b. $-x^2$;
   c. $(2c)^2$;
   d. $2c^2$;
   e. $(-mn)^4$
   f. $-mn^4$
   g. $(a+3b)^2$
   h. $a+3b^2$
   i. $(2x-y)^3$
   j. $2x-y^3$

   a. $(-n)^3=(-n)\cdot(-n)\cdot(-n)$
   b. $-x^2=-x\cdot x$
   c. $(2c)^2=(2c)\cdot(2c)$
   d. $2c^2=2\cdot c\cdot c$
   e. $(-mn)^4=(-mn)\cdot(-mn)\cdot(-mn)\cdot(-mn)$
   f. $-mn^4=-m\cdot n\cdot n\cdot n\cdot n$
   g. $(a+3b)^2=(a+3b)\cdot(a+3b)$
   h. $a+3b^2=a+3b\cdot b$
   i. $(2x-y)^3=(2x-y)\cdot(2x-y)\cdot(2x-y)$
   j. $2x-y^3=2x-y\cdot y\cdot y$

3. Simplify the following expressions:

a) $\frac{2a-ab}{2x-bx}$;

b) $\frac{am-mp}{mn}$;

c) $\frac{25k+5ka}{5a}$;

d) $\frac{14+7k}{7a+49}$;

a. $\frac{2a-ab}{2x-bx} = \frac{a(2-b)}{x(2-b)} = \frac{a}{x}$

b. $\frac{am-mp}{mn} = \frac{m(a-p)}{mn} = \frac{a-p}{n}$

c. $\frac{25k+5ka}{5a} = \frac{5(5k+ka)}{5a} = \frac{5k+ka}{a}$

d. $\frac{14+7k}{7a+49} = \frac{7(2+k)}{7(a+7)} = \frac{2+k}{a+7}$

4. Mary and Jenny have 11 pieces of candy, Jenny and Lisa have 13 pieces of candy and Mary and Lisa have 12 pieces of candy. How many candies they have altogether?

Mary has M candies,
Jenny has J candies,
Lisa has L candies.

$M + J = 11$
$J + L = 13$
$M + L = 12$
E can add together all left parts of the equations and all right parts, the resalting expression will also have left side equal to the right side:
$M + J + J + L + M + L = 36 \;\Rightarrow\; 2M + 2J + 2L = 36.$
$2(M + J + L) = 36$
$M + J + L = 18$
The system of these three equations can be solved in a different way:

$M + J = 11$ $J + L = 13 \quad \Rightarrow$ $M + L = 12$	$M = 12 - L$ $J = 13 - L$	$M + J = 12 - L + 13 - L = 11$ $25 - 2L = 11$ $14 = 2L$ $L = 7, M = 5, J = 6; \quad L + M + J = 18$

5. Simplify the following expression (combine like terms, think about which terms you can add together and which you can't):

$$\left(\frac{1}{7}klm^2 - \frac{4}{3}kl^2m + 7klm\right) + \left(-\frac{3}{21}klm^2 + \frac{4}{9}kl^2m - 5klm\right);$$

$$\left(\frac{1}{7}klm^2 - \frac{4}{3}kl^2m + 7klm\right) + \left(-\frac{3}{21}klm^2 + \frac{4}{9}kl^2m - 5klm\right) =$$
$$= \frac{1}{7}klm^2 - \frac{4}{3}kl^2m + 7klm - \frac{3}{21}klm^2 + \frac{4}{9}kl^2m - 5klm =$$
$$= \frac{1}{7}klm^2 - \frac{3}{21}klm^2 - \frac{4}{3}kl^2m + \frac{4}{9}kl^2m + 7klm - 5klm =$$
$$= \frac{3}{21}klm^2 - \frac{3}{21}klm^2 - \frac{12}{9}kl^2m + \frac{4}{9}kl^2m + 2klm$$
$$= -\frac{8}{9}kl^2m + 2klm$$

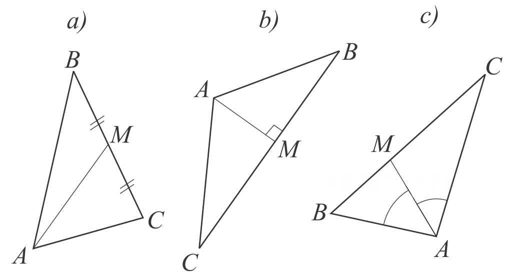
6. Solve the equation (remember, both sides of equation can be multiply/divide by the same number, as well as the same number can be added/subtract to/from both sides):

a) $\frac{5x-4}{2} = \frac{16x+1}{7}$;

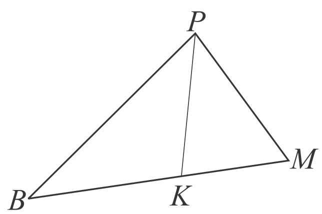
b) $\frac{1-9y}{5} = \frac{19+3y}{8}$;

$\frac{5x-4}{2} = \frac{16x+1}{7}$ $7 \cdot (5x-4) = 2 \cdot (16x+1)$ $35x - 28 = 32x + 2$ $35x - 32x = 2 + 28$ $3x = 30$ $x = 10$ $\frac{5 \cdot 10 - 4}{2} = \frac{16 \cdot 10 + 1}{7}$ $23 = 23$	$\frac{1-9y}{5} = \frac{19+3y}{8}$ $8 \cdot (1-9y) = 5 \cdot (19+3y)$ $8 - 72y = 95 + 15y$ $-72y - 15y = 95 - 8$ $-87y = 87$ $y = -1$ $\frac{1-9\cdot(-1)}{5} = \frac{19+3\cdot(-1)}{8}$ $\frac{1+9}{5} = \frac{19-3}{8}$ $2 = 2$

7. On which picture on the right the segment AM is an altitude, a median, or a bisector?
   a) AM is a median
   b) AM is an altitude
   c) AM is a bisector



8. In a triangle BPM, segment PK is a bisector of the angle BPM. What is the measure of the angle BPK, if the sum of 2 other angles is 80°?

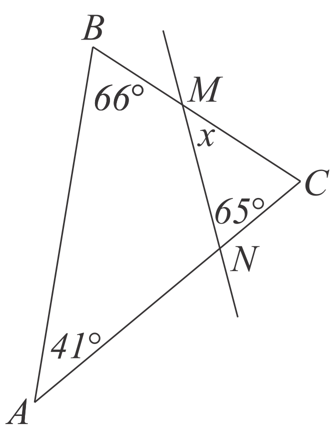


The measure of the angle $\angle$BPM is

$$\angle BPM = 180° - 80° = 100°$$

PK is a bisector, therefore $\angle BPK = \angle KPM = \frac{1}{2}\angle BPM = 50°$

9. What is the measure of the angle $\angle NMC$ (x) on the picture below?
   $\angle BAC \ = 41°, \angle ABM = 66°, \angle MNC = 65°$



$$\angle ACB = 180 - (41 + 66) = 180 - 107 = 73°$$

$$\angle NMC = \angle x = 180 - (65 + 73) = 180 - 138 = 42°$$

10. Which of the following expression is divisible by 9 (check the divisibility by 9 rule)?

1) $\mathbf{151 \cdot 45 + 151 \cdot 36}$
2) $\mathbf{154 \cdot 121 + 815 \cdot 121}$
3) $\mathbf{872 \cdot 45 - 872 \cdot 25}$
4) $\mathbf{574 \cdot 85 - 574 \cdot 65}$

$151 \cdot 45 + 151 \cdot 36 = 151 \cdot 5 \cdot 9 + 151 \cdot 4 \cdot 9 = 9 \cdot (151 \cdot 5 + 151 \cdot 4)$