## Algebra.

## Exercises.

1. Simplify the following expressions:
a. $2+3 a+x y+4-a+x y-6=$
b. $d-4+t+t+32+3 d=$
c. $x+5 s-3 s+2 x=$
d. $4 x-(3 x+(2 x+1))=$
e. $y-(2 y-(3 y-4))=$
f. $z-(2 z+(3 z-(4 z+5)))=$
2. 

a. Julia, Mary and Kate got 51 chocolate bars on Trick-o-treat altogether. Julia got twice as many as

Mary, and Kate got 3 chocolate bars more than Mary. How many candies did each of them get? Write an equation to solve the problem.
3. Simplify the following fractions:

$$
\begin{aligned}
& \frac{12 \cdot 5+12 \cdot 9}{12 \cdot 21}= \\
& \frac{14 \cdot 5-14 \cdot 2}{28}= \\
& \frac{8 \cdot 8-8 \cdot 7}{8 \cdot 5}= \\
& \frac{19 \cdot 8-19 \cdot 6}{38}=
\end{aligned}
$$

## Positive and negative numbers.

Negative numbers represent opposites. If positive represents movement to the right, negative represents movement to the left. If positive represents above sea level, then negative represents below sea level. If positive represents a deposit, negative represents a withdrawal. They are often used to represent the magnitude of a loss or deficiency. Negative numbers appeared for the first time in history in the Nine Chapters on the Mathematical Art, which in its present form dates from the period of the Chinese Han Dynasty ( 202 BC - AD 220), but may well contain much older material. Liu Hui (c. 3rd century) established rules for adding and subtracting negative numbers. By the 7th century, Indian mathematicians such as Brahmagupta were describing the use of negative numbers. Islamic mathematicians further developed the rules of subtracting and multiplying negative numbers and solved problems with negative coefficients. Western mathematicians accepted the idea of negative numbers by the 17th century. Prior to the concept of negative numbers, mathematicians such as Diophantus considered negative solutions to problems "false" and equations requiring negative solutions were described as absurd

Last year when we discuss the negative numbers we used very clear analogy of a basket with balloons and sand bags.

At the beginning basket has N balloons and N sand bags, placed at 0 position and doesn't move. Balloons represent positive numbers, sand bags represent negative numbers. If we add one balloon the basket will move one unit up. If we add one sand bag basket will move one unit down. If we remove one balloon, basket will go one unit down, which is equivalent of adding one sand bag. So $-(+1)=+(-1)$. If we remove one sand bag, basket will go one unit up, which is equivalent of adding one balloon. So $-(-1)=+(+1)$. Let's move to number line:
addition of negativ number
$\xrightarrow{\text { addition of positive number }}$

substraction of positive number substraction of negativ number


Two numbers that have the same magnitude but are opposite in signs are called opposite numbers.

## Multiplication and division of negative numbers.

If we multiply 2 positive numbers we will get third positive number. What will happen if we multiply one negative and one positive number. Let's again review our analogy. In this case we will add or remove our balloons and sand bags by groups of three. Addition of to groups of 3 sand bags will drive the basket 6 units down, because we add 6 bags. So $2 \times(-3)=-6$. (We know that $-1+$ $(-1)+(-1)=-3)$

Removing of 2 groups of 3 sand bags will drive our basket 6 units up, which is corresponding of adding 6 balloons, so $-2 \times(-3)=6$

Addition of 6 balloons ( 2 groups of three) of cause will help us to move up for 6 units. If we remove 2 groups of 3 balloons we will descend 6 units. $-2 \times(+3)=-6$.


## Geometry.

Congruent and non-congruent segments. Two segments are congruent if they can be laid on onto the other so that their endpoints coincide. Suppose that we put the segment [AB] onto the segment [CD] ( pict. below) by placing the point $A$ at the point $C$ and aligning the ray $[A B)$ with the ray $[C D)$. If, as the result of this, the points $B$ and $D$ merge, then the segments $[A B\}$ and $[C D]$ congruent, or equal. Otherwise they are not congruent, and the one which makes a part of the other is considered smaller.


We can introduce the concept of sum of several segments, we can subtract one segment from another.
How to construct the segment equal to another segment?


Draw the segment, equal to the given segment using a compass and a ruler (straight edge):


1. Mark any point on a plane.
2. Open the compass on length of the given segment.
3. Draw the part (or the whole circle).
4. Connect the marked point and any point of the circle.

How to construct the triangle with sides equal to the given segments:


In the very similar way we can define when two angles are congruent. Two angles are congruent if by moving one of them it is possible to identify it with the other.


