## Algebra.

## 1. Equalities: equations and identities.

## Expressions.

Mathematical expressions are the mathematical phrases that contain numbers, symbols, letters. Terms can be numbers or numbers combined with letters. In the latter case letters are called "variables" and a number is called "coefficient". If the
 term contains only the number than it's called "constant".

In the term $2 a b$ number 2 is a coefficient and $a$, and $b$ are variables. The "like terms" in the expression above are ones that have the same variable. All constants are like terms as well. To simplify the expression all like terms should be combined. In other words, all constant should be added together as well as all terms which contain the same variables. For the expression above

$$
\begin{aligned}
3 x-2 a b-10+4 x+24-5 a b+75 y & =3 x+4 x-2 a b-5 a b+75 y-10+24= \\
= & 7 x-7 a b+75 y+14
\end{aligned}
$$

Is there any difference between two following equalities?
$a(b+c)=a b+a c$
$a+2=6$

Letters $a, b$, and $c$ in both these expressions are called variables, we can put any number (whole or fraction) into it. In the first case the equality is still a true expression for any $a, b$, and $c$, this is a distributive property of addition.
The second expression is a true expression for only one value of $a=4$ and we call this kind of expressions "an equation". An equation is the problem of finding values of some variables, called unknowns, for which the specified equality is true. We have to solve the equation to find the value of an unknown variable.

## 2. How to solve an equation?

An equation is a statement that the values of two mathematical expressions are equal (indicated by the $\operatorname{sign}=$ ). For example, in the equation

$$
3 x-5=4 x-7
$$

one expression $(3 x-5)$ equal to the expression $(4 x-7)$. Solving the equation, means to find such number $x$ that will make the equality true.

In order to do it first we have to combine all like terms of the expressions. Because both side of the equation are equal than the equal terms can be added (or subtract) to (from) both sides and it will not change the equality rule:

$$
\begin{gathered}
3 x-5=4 x-7 \\
3 x-3 x-5=4 x-3 x-7 \\
-5=x-7 \\
-5+7=x-7+7 \\
2=x
\end{gathered}
$$

It is not really necessary to write all this sequential statements, we just need to rewrite the term on another side of the equation with the opposite sign (but you have to know why this is the right way to do). Both sides of the equation can be divided (or multiplied) by the same number (or term) and as the result we will get the equality again.

$$
\begin{aligned}
& 4 \cdot(x+5)=12 \\
& \frac{4 \cdot(x+5)}{4}=\frac{12}{4} \\
& x+5=3 \\
& x+5-5=3-5 \\
& x=-2
\end{aligned}
$$

## Exercises.

1. Simplify the following expressions:
a. $2+3 a+x y+4-a+x y-6=$
b. $d-4+t+t+32+3 d=$
c. $x+5 s-3 s+2 x=$
2. Solve the following equations:

$$
x \cdot \frac{3}{5}=\frac{2}{5}
$$

$$
3 y+\frac{1}{2}=y+\frac{3}{2}
$$

$$
\frac{1}{2} z+\frac{3}{4}=\frac{3}{2} z-\frac{1}{4}
$$

$$
d \div \frac{2}{3}+\frac{1}{2}=\frac{7}{8}
$$

3. Solve the following equations for $x$ ( $x$ is the variable, $\mathrm{a}, \mathrm{b}, \mathrm{c}$, and d are parameters):
a. $a \cdot x+b=c$
b. $b \cdot x-d=a+b$
4. Can you find $x$, and $y$ which satisfy all three relations below?

$$
\left\{\begin{array}{c}
x+y=10 \\
x-y=4
\end{array}\right.
$$

5. On the first shelf there are 6 fewer books than on 2 others altogether. On the second shelf there are 10 fewer books than om 2 others. How many books are there on the third shelf?
6. Which 1-digit numbers are represented by letters in the following problem?
$\begin{array}{r}A \\ +A B \\ A B C \\ \hline B C B\end{array}$
7. One of the angles of the triangle is 2 times smaller than the second angle and three times smaller than the third angle. Find the angles of this triangle. Remember, that three angles of any triangle add up to a straight angle $\left(180^{\circ}\right)$.
8. Lisa is 8 years older than Mary. 2 years ago, she was 3 times as old as Mary. How old both girls are?

## Geometry.



Eight angles of a transversal. (Vertical angles such as $\gamma$ and $\alpha$ are always congruent.)


Transversal between nonparallel lines. Consecutive angles are not supplementary.


Transversal between parallel lines. Consecutive angles are supplementary.

Transversal
Parallel Lines
Vertical Angles
Corresponding Angles
Alternate Interior Angles
Alternate Exterior Angles Consecutive Interior Angles

We can reformulate Euclid's fifth postulate,
 in this way

If 2 consecutive interior angles are supplementary (add up to a straight angle), then these 2 lines are parallel. (if they add up to an angle less than straight angle, these 2 line will intersect on this side). This postulate will be considering a true statement without proof.

Congruent and non-congruent segments. Two segments are congruent if they can be laid on onto the other so that their endpoints coincide. Otherwise they are not congruent, and the one which makes a part of the other is considered smaller.

How to construct the segment equal to another segment?
Draw the segment, equal to the given segment using a compass and a ruler (straight edge):


1. Mark any point on a plane.
2. Open the compass on length of the given segment.
3. Draw the part (or the whole circle).
4. Connect the marked point and any point of the circle.

How to construct the triangle with sides equal to the given segments:


