Accelerated Math. Class work 4.

Algebra.

Divisibility rules.

Divisibility Rules	
A number is divisible by	….. if and only if
2	If last digit is 0, 2, 4, 6, or 8
3	If the sum of the digits is divisible by 3
4	If the last two digits is divisible by 4
5	If the last digit is 0 or 5
6	If the number is divisible by 2 and 3
7	cross off last digit, double it and subtract. Repeat if you want. If new number is divisible by 7, the original number is divisible by 7
8	If last 3 digits is divisible by 8
9	If the sum of the digits is divisible by 9
10	If the last digit is 0
11	Subtract the last digit from the number formed by the remaining digits. If new number is divisible by 11, the original number is divisible by 11
12	If the number is divisible by 3 and 4

Factorization.

In mathematics factorization is a decomposition of on object into a product of other objects, or representation of an object as a product of 2 or more objects, which called 'factors'. For example, we can represent the expression $a \times b + a \times c$ as a product of a and expression $(b + c)$.

$$a \times b + a \times c = a \times (b + c)$$

Or in a numerical expression:

$$7 \times 5 + 7 \times 3 = 7 \times (5 + 3)$$

Or a number can be representing as product of two or more other numbers, for example:

$$40 = 4 \times 10, \quad 36 = 6 \times 6$$

Does any natural number can be represented as a product of 2 or more numbers besides 1 and itself? Natural numbers greater than 1 that has no positive divisors other than 1 and itself are called prime numbers.

Even numbers are the numbers divisible by 2 (they have 2 as a divisor), so they can be factorized as 2 times something else. Can an even number be a prime number? Is there any even prime number?

Prime factorization or integer **factorization** of a number is the determination of the set of **prime** numbers which multiply together to give the original integer. It is also known as **prime** decomposition.

Prime factorization process:

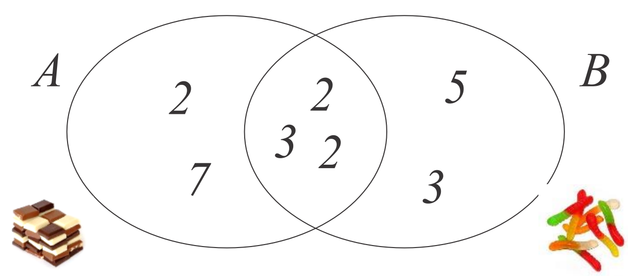
$$\begin{array}{r|l} 168 & 2 \\ 84 & 2 \\ 42 & 2 \\ 21 & 3 \\ 7 & 7 \\ 1 & \end{array} \qquad \begin{array}{r|l} 180 & 2 \\ 90 & 2 \\ 45 & 3 \\ 15 & 3 \\ 5 & 5 \\ 1 & \end{array}$$

Prime factors of 168 are $2, 2, 2, 3, 7$ and prime factors of 180 are $2, 2, 3, 3, 5,$

$$2 \times 2 \times 2 \times 3 \times 7 = 168; \quad 2 \times 2 \times 3 \times 3 \times 5 = 180$$

Any natural number has single unique prime factorization.

For Halloween the Jonson family bought 168 mini chocolate bars and 180 gummi worms. What is the largest number of kids between whom the Jonsons can divide both kinds of candy evenly?



To solve this problem, we have to find a number which can serve as a divisor for 168 as well as for 180. There are several such numbers. The first one is 2. Both piles of candy can be evenly divided between just 2 kids. 3 is also a divisor. The Jonson family wants to treat as many kids as possible with equal numbers of candy. To do this they have to find the Greatest Common Divisor (GCD), the largest number that can be a divisor for both (168 and 180) amounts of candy. Let's take a look at a set of all prime factors of 168 and 180. For 168 this set contains 2, 2, 2, 3, and 7. Any of these numbers as well as any of their products divides into 168. The same goes for the set of prime factors of 180, which are 2, 2, 3, 3, and 5. It is easy to see that these two sets have common elements. It means that both numbers are divisible by any of these common elements and any their products. The largest product is the product of all common elements. This largest product is GCD.

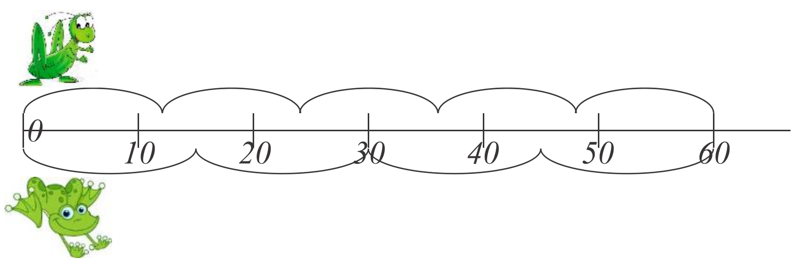
$$168 \div 12 = 14$$

$$180 \div 12 = 15$$

Between 12 kids they can divide both kinds of candy evenly.

A grasshopper jumps a distance of 12 centimeters each jump. A little frog jumps a distance of 15 centimeters each jump. They start hopping at the same time from the same point 0 and jump along the big ruler. What is the closest point on the ruler at which they can meet?

There are places on the ruler that both of them can reach after some number of jumps. One of such places is, of course, 12×15 cm. A grasshopper can make 15 jumps while a little frog can make only 12 jumps. Will 12x15 be the only place where they can meet or there are some other places? If this is the case, we have to find a number that is divisible by both 12 and 15. Take into account that 12×15, as well as any product of 12×15 is divisible by both 12 and 15.

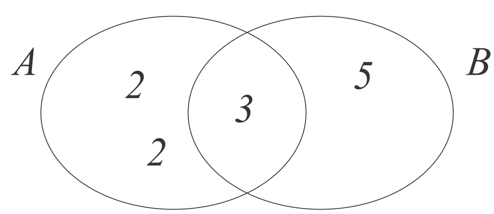


Is there are any number which is less then 12×15 and still divisible by 12 and 15? Prime factorization of 12 and 15:

$$\begin{array}{r|l} 12 & 2 \\ 6 & 2 \\ 3 & 3 \\ 1 & \end{array} \qquad \begin{array}{r|l} 15 & 3 \\ 5 & 5 \\ 1 & \end{array}$$

$$12 \times 15 = (2 \times 2 \times 3) \times (3 \times 5)$$

The number which we are looking for has to be a product of prime factors of either 12 or 15, so it should be a union of two sets – set of prime factors of 12 and 15.



$$2 \times 2 \times 3 \times 5 = 60$$

60 is the smallest number, which is divisible by 12 and 15, LCM.

Eratosthenes proposed a simple algorithm for finding prime numbers. This algorithm is known in mathematics as the Sieve of Eratosthenes.

In mathematics, the sieve of Eratosthenes, one of a number of prime number sieves, is a simple, ancient algorithm for finding all prime numbers up to any given limit. It does so by iteratively marking as composite, *i.e.*, not prime, the multiples of each prime, starting with the multiples of 2.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Exercises.

1. Can the expression below be a true statement, if letters are replaced with numbers from 1 to 9 (different letters correspond to different numbers).

$$f \cdot l \cdot y = i \cdot n \cdot s \cdot e \cdot c \cdot t$$

2. The product of three digits of three-digit number ABB equals to two-digit number AC, the product of these two digits equal C. What is this tree digit number?
3. Odd or even number is the sum

$$1 + 2 + 3 + 4 + \cdots + 99 =$$

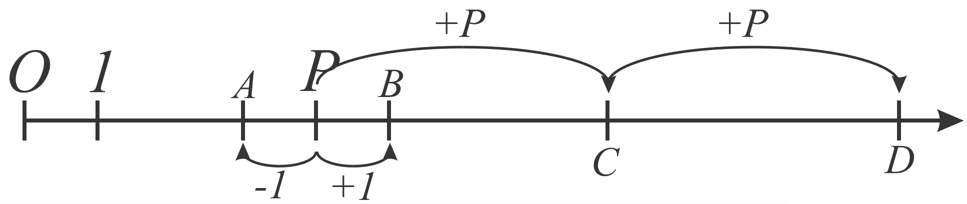
4. Proof that
   a. Sum of any number of even terms is even
   b. Sum of even number of odd terms is even
   c. Sum of odd number of odd terms is odd
5. Two buses leave from the same bus station following two different routes. For the first one it takes 48 minutes to complete the roundtrip route. For the second one it takes 1 hour and 12

minutes to complete the round trip route. How much time will it take for the buses to meet at the bus station for the first time after the have departed for their routes at the same time?

6. A florist has 36 roses, 90 lilies, and 60 daisies. What is largest amount of bouquets he can create from these flowers evenly dividing each kind of flowers between them?

7. Mary has a rectangular backyard with sides of 48 and 40 yards. She wants to create square flower beds, all of equal size, and plant different kind of flowers in each flower bed. What is the largest possible size of her square flower bed?

8. On a number line we marked numbers A, B, C, D. Can numbers A, B, C, D be prime numbers if number P is a prime number? Explain you answer. Can three consecutive numbers be prime numbers?



9. Prove that for 2 natural numbers a and b GCD(a, b)· LCM(a, b) = $a \cdot b$

10. John goes to the movie theater every third day, Robert goes to the theater every 5th day and Peter goes to the movie theater every 7th day. Today they went to see a movie together. When they will meet at the movie theater next time?

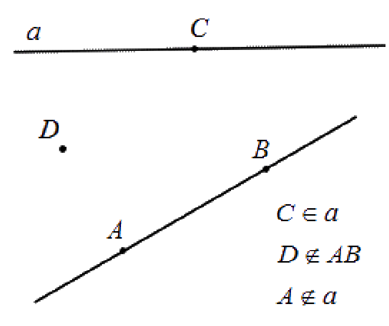
11. Compute:

a. $\left(\frac{1}{3}+\frac{2}{9}\right) \div \left(\frac{9}{10}-\frac{2}{5}\right) =$

b. $\left(4-\frac{2}{3}\right) \times \left(1\frac{1}{2}-\frac{3}{4}\right) =$

c. $\frac{7}{16}+\frac{9}{10} \times \frac{5}{14} \times \frac{7}{12} =$

d. $1-\frac{9}{16} \div \frac{9}{4}-\frac{1}{12} =$

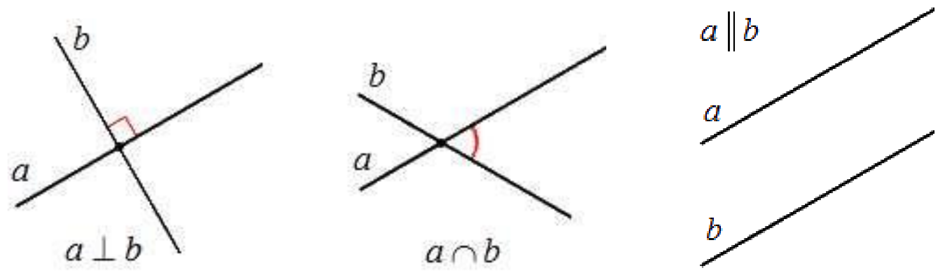


Geometry.

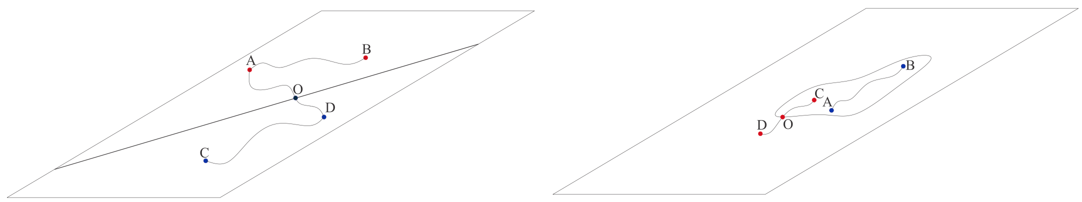
Points, lines, and plane.

There are two possibilities of mutual location of the line and the point on the plane: a point lies on a line or a point doesn't lie on the straight line. If 2 lines have 2 common points these lines coincide. Two straight line can intersect (then they have one common point) or they can be parallel.

Parallel lines are lines in a plane which do not meet; that is, two lines in a plane that do not intersect or touch each other at any point are said to be parallel.



Each straight line divides a plane into two domains. In these domains any two points on one side of the line may be connected without crossing the line itself and any two points on the two different sides of the lane can't be connected without crossing the line.



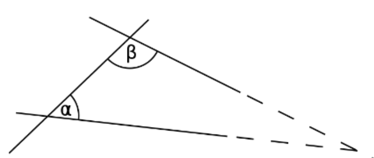
Enclosed area on a plane is the area limited by a closed curved line (or chain of line segments) any 2 points of which can be connected without crossing the curved line (or series of line segments) and any point inside of the limit can't be connected with any point outside of the limit without crossing the curved line (or chain of line segments).

Two of the most important building blocks of geometric proofs are axioms and postulates. Axioms and postulates are essentially the same thing: mathematical truths that are accepted without proof. Their role is very similar to that of undefined terms: they lay a foundation for the study of more complicated geometry. One of the greatest Greek (*Euclid*) achievements was setting up such rules for plane geometry. This system consisted of a collection of undefined terms like point and line, and five axioms from which all other properties could be deduced by a formal process of logic. Four of the axioms were so self-evident that it would be unthinkable to call any system a geometry unless it satisfied them:

1. A straight line may be drawn between any two points.
2. Any terminated straight line may be extended indefinitely.
3. A circle may be drawn with any given point as center and any given radius.
4. All right angles are equal.

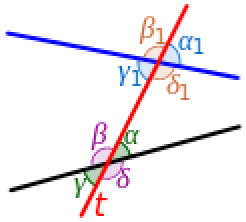
But the fifth axiom was a different sort of statement:

5. If two straight lines in a plane are met by another line, and if the sum of the internal angles on one side is less than two right angles, then the straight lines will meet if extended sufficiently on the side on which the sum of the angles is less than two right angles.

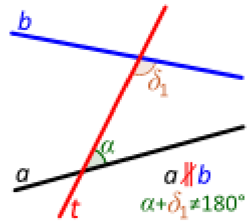


Mathematicians found alternate forms of the axiom that were easier to state, for example:

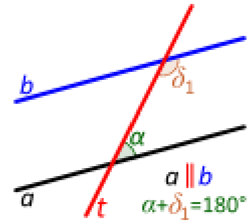
5'. For any given point, not on a given line, there is exactly one line through the point that does not meet the (or parallel to) given line.



Eight angles of a transversal. (Vertical angles such as γ and α are always congruent.)



Transversal between non-parallel lines. Consecutive angles are not supplementary.



Transversal between parallel lines. Consecutive angles are supplementary.

The Euclid's *Elements* also include the following five "common notions":

1. Things that are equal to the same thing are also equal to one another (formally the Euclidean property of equality, but may be considered a consequence of the transitivity property of equality).
2. If equals are added to equals, then the wholes are equal (Addition property of equality).
3. If equals are subtracted from equals, then the remainders are equal (Subtraction property of equality).
4. Things that coincide with one another are equal to one another (Reflexive Property).
5. The whole is greater than the part.

Exercises.

1. How it can be that two straight lines do not intersect but they are not parallel?
2. A goat is tied to a stake (or 2 poles) with a rope of length (L). What shape it will graze?