Accelerated Math. Class work 3.

Algebra.

We say that a natural number is divisible by another natural number if the result of this operation is a natural number. If this is not the case then we can divide a number with a remainder.

$a:b=c$

dividend divisor quotient

$a=b\cdot c+r$

dividend divisor quotient remainder

If a and n are natural numbers, the result of division operation of $a \div n$ will be a quotient c, such that

$$a = b \times c + r$$

Where r is a remainder of division $a \div b$. If r is 0, then we can tell that a is divisible by b.

- If we want to divide m by 15, what numbers we can get as a remainder?

If the remainder is 0, then quotient and divisor are both factors of dividend, $a = b \cdot c$, and to divide a number $\boldsymbol{a}$ by another number, $\boldsymbol{b}$, means to find such number $\boldsymbol{c}$, that $c \cdot b$ will give us a. So, because the product of 0 and any number is 0, than there is no such arithmetic operation as division by 0.

Divisibility rules.

Divisibility Rules	
A number is divisible by	….. if and only if
2	If last digit is 0, 2, 4, 6, or 8
3	If the sum of the digits is divisible by 3
4	If the last two digits is divisible by 4
5	If the last digit is 0 or 5
6	If the number is divisible by 2 and 3
7	cross off last digit, double it and subtract. Repeat if you want. If new number is divisible by 7, the original number is divisible by 7
8	If last 3 digits is divisible by 8
9	If the sum of the digits is divisible by 9
10	If the last digit is 0
11	Subtract the last digit from the number formed by the remaining digits. If new number is divisible by 11, the original number is divisible by 11
12	If the number is divisible by 3 and 4

A statement (or proposition) is a sentence that is either true or false, but not both. So '3 is an odd integer' is a statement. But 'π is a cool number' is not a (mathematical) statement. Note that '4 is an odd integer' is also a statement, but it is a false statement.

Are these sentences statements or not? If yes, are they true or false? Can you prove it?

- Telephone numbers in the USA have 10 digits.
- The moon is made of cheese.
- The sum of 2 odd natural number is an even number
- Would you like some cake?
- $3 + x = 12$
- The sum of two even numbers.
- $1 + 3 + 5 + 7 + \cdots + 2n + 1.$
- Go to your room!
- $7 + 3 = 10$
- All birds can light.

The rule of divisibility by 2 is:

If the last digit of a number is an even number or 0 (0, 2, 4, 6, or 8) the number is even number (divisible by 2).

Proof of the divisibility by 2 rule:

Any natural number can be written as a sum:

$$\ldots + 1000 \times n + 100m + 10 \times l + k = \cdots + 2 \times 500 \times n + 2 \times 50 \times m + 2 \times 5 \times l + k$$

Were n, m, l, and k are number of thousands, number of hundreds, number of tens, and number of units. If *k* is an even number, it also can be represented as a product of 2 and something else. Then the number can be written as:

$$\ldots + 1000 \times n + 100m + 10 \times l + k = \cdots + 2 \times 500 \times n + 2 \times 50 \times m + 2 \times 5 \times l + 2 \times p$$

(*p* can be 0, 1, 2, 3, and 4. Do you know why?). Distributive property is allowing us to represent this expression as a product:

$$\ldots + 1000 \times n + 100m + 10 \times l + k = \cdots + 2 \times 500 \times n + 2 \times 50 \times m + 2 \times 5 \times l + k$$
$$= 2 \times (\ldots + 500 \times n + 50 \times m + 5 \times l + p)$$

Now we can see that the number is divisible by 2 if its last digit is even or 0.

All other divisibility rules can be proved as well.

Exercises:

1. Proof that the sum of 2 odd natural number is an even number.
2.
   a. The remainder of 1932 ÷ 17 is 11, the remainder of 261 ÷ 17 is 6. Is 2193 = 1932 + 261 divisible by 17? Can you tell without calculating and dividing?

   b. Find all natural numbers such that upon division by 7 they give equal quotient and remainder.
3. Even or odd number will be the sum and the product of
   a. 2 odd numbers
   b. 2 even numbers
   c. 1 even and 1 odd number
   d. 1 odd and 1 even number

   Can you explain why?
4. Can the expression below be a true statement, if letters are replace with numbers from 1 to 9 (different letter correspond to different number).

$$a \cdot b \cdot c = d \cdot e \cdot f \cdot g \cdot h \cdot i$$

5. There are red, green, and blue pencils in the box, 20 pencils altogether. There are 6 times as many blue pencils as the green ones, there are fewer red pencils then blue pencils. How many red, green, and blue pencils are there in the box?
6. There are red and blue balloons in the room, 85 balloons altogether. At least one of them is red. In any random pair of the balloons at least one is blue. How many red and how many blue balloons are there in the room?
7. Peter is always saying the truth, Alex is always lying. What question you have to ask to get the same answer from both boys?
8. The product of three digits of three-digit number ABB equals to two-digit number AC, the product of these two digits equal C. What is this tree digit number?

Factorization.

In mathematics factorization is a decomposition of on object into a product of other objects, or representation of an object as a product of 2 or more objects, which called ‘factors’. For example, we can represent the expression $a \times b + a \times c$ as a product of a and expression $(b + c)$.

$$a \times b + a \times c = a \times (b + c)$$

Or in a numerical expression:

$$7 \times 5 + 7 \times 3 = 7 \times (5 + 3)$$

Or a number can be representing as product of two or more other numbers, for example:

$$40 = 4 \times 10, \quad 36 = 6 \times 6$$

Does any natural number can be represented as a product of 2 or more numbers besides 1 and itself? Natural numbers greater than 1 that has no positive divisors other than 1 and itself are called prime numbers.

Even numbers are the numbers divisible by 2 (they have 2 as a divisor), so they can be factorize as 2 times something else. Can an even number be a prime number? Is there any even prime number?

Prime factorization or integer **factorization** of a number is the determination of the set of **prime** numbers which multiply together to give the original integer. It is also known as **prime** decomposition.

$$\begin{array}{r|l} 168 & 2 \\ 84 & 2 \\ 42 & 2 \\ 21 & 3 \\ 7 & 7 \\ 1 & \end{array} \qquad \begin{array}{r|l} 180 & 2 \\ 90 & 2 \\ 45 & 3 \\ 15 & 3 \\ 5 & 5 \\ 1 & \end{array}$$

Prime factorization process:

Prime factors of 168 are $2, 2, 2, 3, 7$ and prime factors of 180 are $2, 2, 3, 3, 5,$

$$2 \times 2 \times 2 \times 3 \times 7 = 168; \quad 2 \times 2 \times 3 \times 3 \times 5 = 180$$

Eratosthenes proposed a simple algorithm for finding prime numbers. This algorithm is known in

mathematics as the Sieve of Eratosthenes.

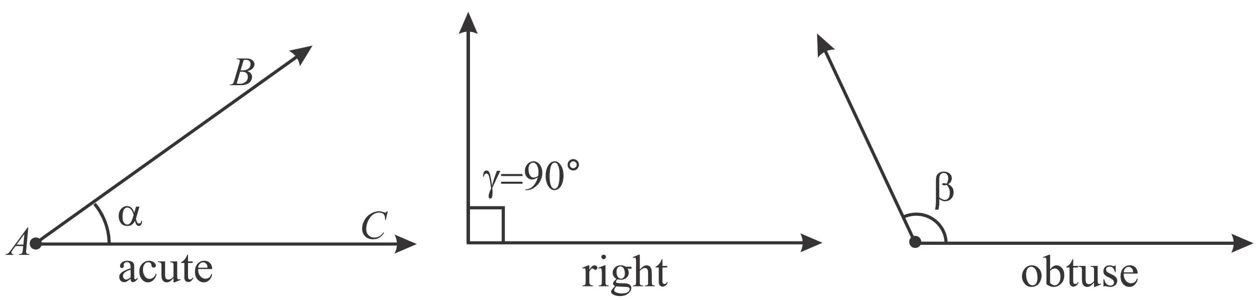
In mathematics, the sieve of Eratosthenes, one of a number of prime number sieves, is a simple, ancient algorithm for finding all prime numbers up to any given limit. It does so by iteratively marking as composite, *i.e.*, not prime, the multiples of each prime, starting with the multiples of 2.

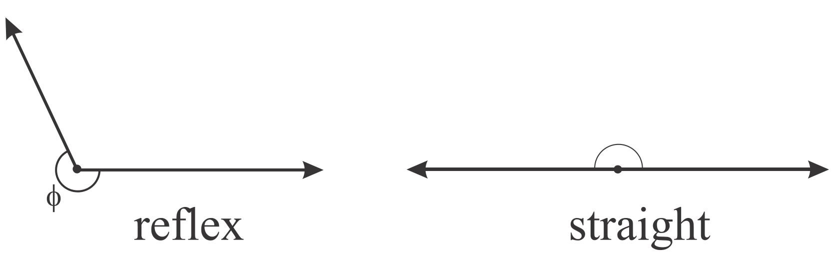
~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Geometry.

An angle is the figure formed by two **rays**, called the sides of the angle, sharing a common endpoint, called the **vertex** of the angle.

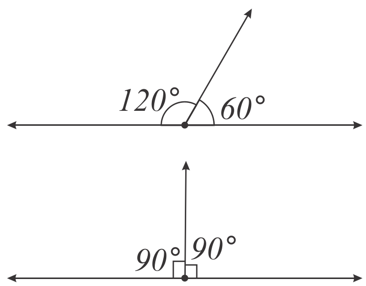
Angles notations are usually three capital letters with vertex letter in the middle or small Greek letter: ∠ABC, α. Measure of the angle is the amount of rotation required to move one side of the angle onto the other. As the angle increases, the name changes:



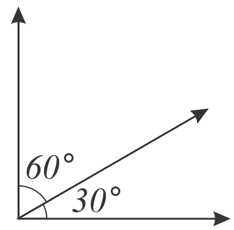


Strait angle is formed by two rays on the same strait line. Straight angle has a measure of 180°.

Two angles are called adjacent if they have common vertex and a common side. If two adjacent angles combined form straight angle they are called supplementary; if they form right angle than they are called complementary.



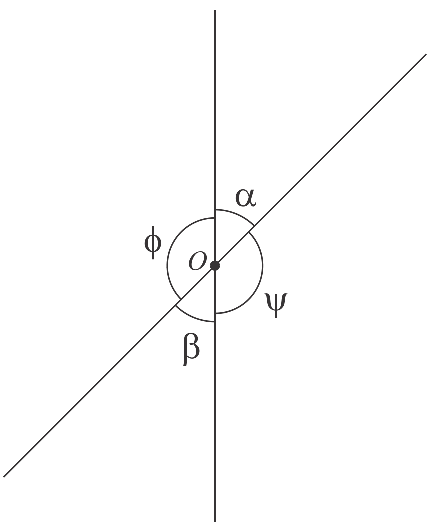
supplementary



complementary

An angle which is supplementary to itself we call right angle. Lines which intersect with the right angle we call perpendicular to each other.

When two straight lines intersect at a point, four angles are formed. A pair of angles opposite each other formed by two intersecting straight lines that form an "X"-like shape, are called vertical angles, or opposite angles, or vertically opposite angles.



α and β and ϕ and ψ are 2 pairs of vertical angles.

***Vertical angles theorem**:*

Vertical angles are equal.

In mathematics, a **theorem** is a statement that has been proven on the basis of previously established statements. According to a historical legend, when Thales visited Egypt, he observed that whenever the Egyptians drew two intersecting lines, they would measure the vertical angles to make sure that they were equal. Thales concluded that one could prove that vertical angles are always equal and there is no need to measure them every time.

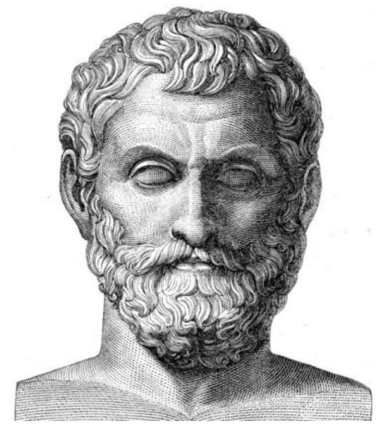
Proof:

$\angle\phi + \angle\alpha = 180°$ because they are supplementary by construction.

$\angle\phi + \angle\beta = 180°$ because they are supplementary also by construction.

$\Rightarrow \angle\alpha = \angle\beta$, therefore we proved that if 2 angles are vertical angles then they are equal. Can we tell that invers is also the truth? Can we tell that if 2 angles are equal than they are vertical angels?

(**Thales of Miletus** 624-546 BC was a Greek philosopher and mathematician from Miletus. Thales attempted to explain natural phenomena without reference to mythology. Thales used geometry to calculate the heights of pyramids and the distance of ships from the shore. He is the first known individual to use deductive reasoning applied to geometry, he also has been credited with the discovery of five theorems. He is the first known individual to whom a mathematical discovery has been attributed (Thales theorem).



Exercises.

9. 4 angles are formed at the intersection of 2 lines. One of them is 30°. What is the measure of 3 others?
10. Draw 2 angles in such way that they intersect

a. by a point

b. by a segment

c. by a ray

d. don't intersect at all.

11. * 3 lines intersect at 1 point and form 6 angles. One is 44°, another is 38°. Can you find all other angles?
12. *Right angle is divided into 3 angles by 2 rays. One of this angles by 20° more than the other and by 20° less the third one. What are the measures of these 3 angles?
13. On the picture below ∠BOD = 152°, ∠COD = 55°, angle ∠AOD is a straight angle. Find the measures of all other angles on the picture.