

Homework 21

Black body radiation

During last class we discussed black body radiation. We learned that any object of nonzero temperature has to emit electromagnetic radiation and to absorb electromagnetic radiation emitted by its surroundings. The object which is hotter than its surroundings emits more than absorbs and cools down. A cold object obtains more energy from surroundings than the object emits and warms up.

We learned that the amount of energy emitted from an object depends on the object material and on its temperature. The higher the object's temperature, the more energy is emitted from the surface per unit time. It turns out that the total power $R(T)$ emitted from a unit surface area of an absolutely black body (the body which is able to emit and absorb electromagnetic radiation of all frequencies) obeys *Stefan-Boltzmann law*:

$$R(T) = \sigma \cdot T^4 \quad (1)$$

Here $\sigma = 5.67 \cdot 10^{-8} \text{ W}/(\text{K} \cdot \text{m}^2)$ is the *Stefan-Boltzmann constant*. As you may remember, another name for $R(T)$ is "radiant exitance". The law is named after Austrian-Slovenian physicist, mathematician and poet Joseph Stefan (1835-1893) and Austrian physicist and philosopher Ludwig Boltzmann (1844-1906).



Joseph Stefan



Ludwig Boltzmann

As the temperature of a black body increases, not only radiant exitance but the spectrum of electromagnetic radiation is changing. The wavelength of maximum emission decreases with increasing temperature so that:

$$w = \lambda_{max} \cdot T = 2898 [\mu\text{m} \cdot \text{K}] \quad (2)$$

Expression (2) is called the *Wien displacement law* and w is the *Wien constant*. It is named after a German physicist Wilhelm Wien (1864-1928).



Wilhelm Wien

Problem.

Assume that the Earth is in thermal equilibrium and radiates energy into space at the same rate at which it receives it from the Sun. At what orbit radius around the Sun would the ocean freeze? How does the answer compare with the actual Earth's orbit radius of $1.5 \cdot 10^{11} \text{ m}$? (Let us assume that the Earth behaves as a black body, the Sun's radiant flux is $3.9 \cdot 10^{26} \text{ W}$ and there is fresh water in the ocean 😊)