Homework 4.

Refraction at a spherical surface.

Let us consider refraction at a spherical surface. We have two media with refractive indices $\mathbf{n_1}$ and $\mathbf{n_2}$. The media are separated with a spherical boundary (Figure 1)

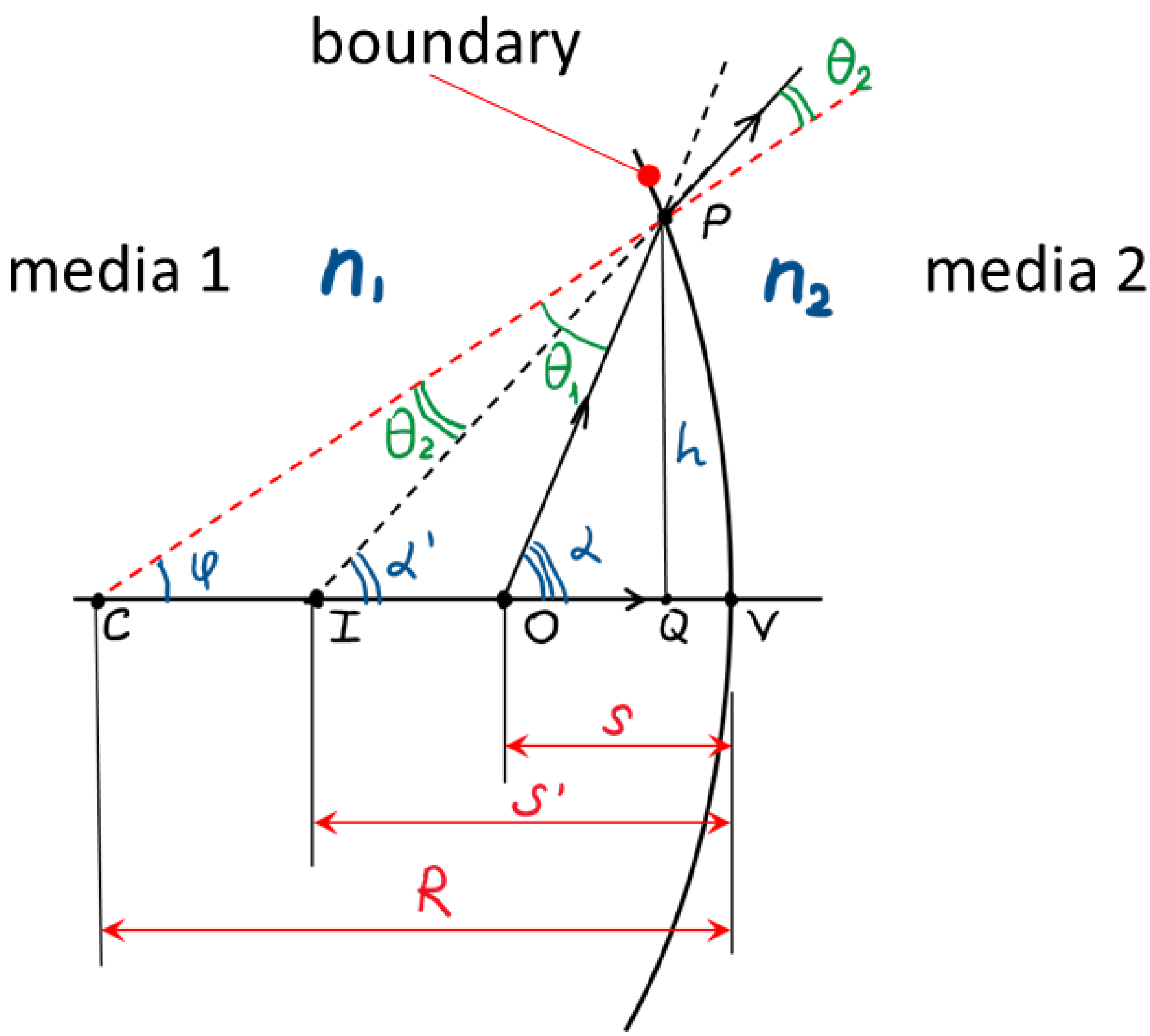

Figure 1. Refraction at a spherical surface.

We are going to find the image of point O. For this we will use two rays emanating from point O: OQ and OP. Ray OP is refracted at point P, the refraction angle is Θ_1. Point C is the center of the spherical surface, so CP is the radius of the sphere. As we know, CP is perpendicular to the sphere's tangent plane in point P. So, for ray OP, $\boldsymbol{\theta_1}$ is the angle of incidence and $\boldsymbol{\theta_2}$ is the angle of refraction. According to Snell's law we have:

$$n_1 sin\theta_1 = n_2 sin\theta_2 \qquad (1)$$

The refracted ray and the ray OQ appear to emerge from their common intersection, point **I** which is the image of point **O**. In triangle CPO, **α** is the exterior angle, so we have:

$$\alpha = \theta_1 + \varphi \qquad (2)$$

For triangle CPI we have:

$$\alpha' = \theta_2 + \varphi \qquad (3)$$

From (1), (2) and (3), we obtain:

$$n_1(\alpha - \varphi) = n_2(\alpha' - \varphi) \qquad (4)$$

Then we will neglect the distance QV (we assume that the radius R is much larger than h) and express the angles **α, α'** and **φ** through **s, s'** and **h** as:

$$\alpha \approx \frac{h}{s};\ \alpha' \approx \frac{h}{s'};\ \varphi \approx \frac{h}{R} \qquad (5)$$

Here we used the approximation $tg(\alpha) \approx \alpha$, which is valid for small angles. So we have:

$$n_1\left(\frac{h}{s} - \frac{h}{R}\right) = n_2\left(\frac{h}{s'} - \frac{h}{R}\right) \qquad (6)$$

or

$$\frac{n_1}{s} - \frac{n_2}{s'} = \frac{n_1 - n_2}{R} \qquad (7)$$

Problems:

1. How does formula (7) change if we will consider convex boundary?
2. Try to find geometrical position of an image of an object produced by a higher refraction index region bounded by convex and concave surfaces (see Fig below)