Homework 6.

Torque

During last class we discussed torque. For rotational dynamics torque plays same role as force plays for the linear dynamics. If a nonzero torque applied to an object, the object experience angular acceleration $\boldsymbol{\beta}$. Simply speaking, to turn an object we have to apply torque to it.

Imagine that you are trying to rotate a rigid body (Fig. 1) with respect to the axis passing through the point A ("pivot point") and perpendicular to the page plane.

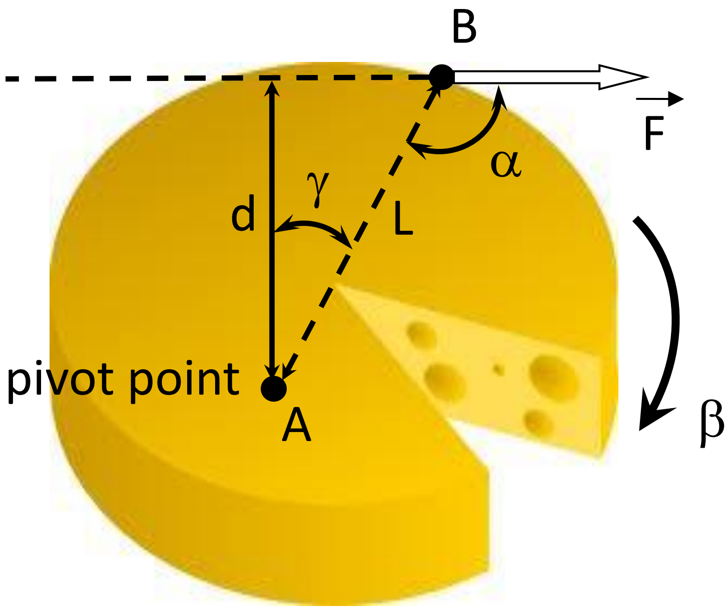


Figure 1.

$$\left|\vec{T}\right| = d \cdot \left|\vec{F}\right| \qquad (1)$$

Torque can be calculated as a product of the force magnitude and the distance ***d*** between the pivot point and the straight line which passes through the point to which the force is applied (point B in Fig.1) and parallel to the force direction. Torque can also be calculated through the distance L between pivot point and the point of the force application. As we can see from the Figure 1, $d = L \cdot \cos\gamma = L \cdot \sin\alpha$. So

$$\left|\vec{T}\right| = \left|\vec{F}\right| \cdot L \cdot \sin\alpha \qquad (2),$$

where $\boldsymbol{\alpha}$ is the angle between the force direction and the line connecting pivot point and point to which the force is applied. Torque is a vector – in a sense that it has both magnitude and direction. Let us consider the sign of the torque as the indication where it "wants" to turn the object – clockwise or counterclockwise. If we know total net torque $\boldsymbol{T_{tot}}$ which applied to the body, we can calculate the angular acceleration $\boldsymbol{\beta}$:

$$\overrightarrow{T_{tot}} = I \cdot \vec{\beta} \qquad (3),$$

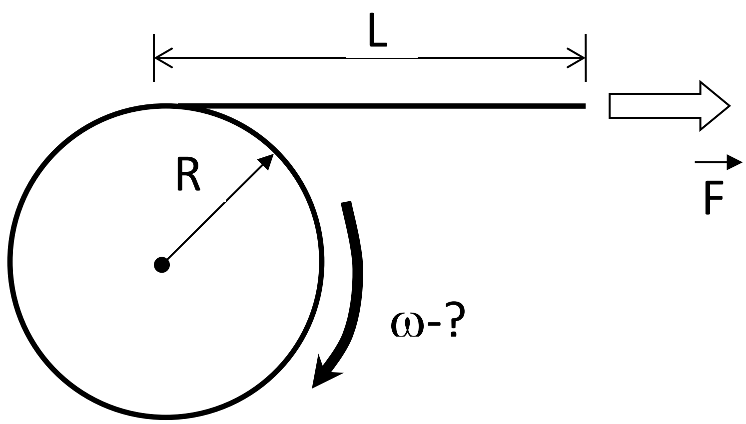
where I is the moment of inertia. *Please note that both torque and the moment of inertia in the formula (3) should be calculated with respect to same axis.* If you use a different axis both torque and moment of inertia can be different. Compare formula (3) with the Newton's second law:

$$\overrightarrow{F_{tot}} = m \cdot \vec{a} \qquad (4),$$

Where $\boldsymbol{m}$ is mass, $\boldsymbol{a}$ – acceleration, $\boldsymbol{F_{tot}}$ – total net force applied to the body.

Problems:

1. A cylindrical coil of a radius R with thread can rotate without friction around the coil's "main' axes. You are pulling the thread with a constant force F. The mass of the coil is *m* (the thread is very light). What is angular velocity of the coil after you pulled out a thread's tail of the length *l?* (See picture below).



2. Find final kinetic energy of the coil.