Homework 6

Rolling

Let us imagine that the center of a rolling wheel is moving along the ground at a constant velocity V. We can see that as the wheel makes a full turn its center is displaced by 2πR where r is the radius of the wheel. The time which is required for the displacement is:

$$t = \frac{2\pi R}{V} \quad (1)$$

According to the definition of angular velocity it is total angle divided by the time which is required to turn to this angle. The angle swept for one full turn is 2p rad. So, the angular velocity is:

$$\omega = \frac{2\pi}{t} = \frac{V}{R} \quad (2)$$

If you walking 'together" with the rolling wheel with the same velocity as its center has, you will see that the wheel is just rotating with the angular velocity described by the formula (2). In the reference frame connected with the center of the rolling wheel the upper and lower points of the wheel will have the same speed but their velocities will have opposite directions (Figure 1, left)

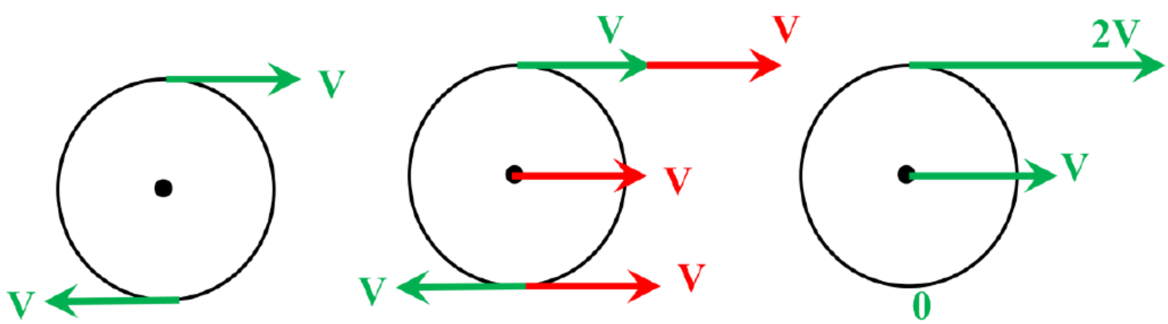


Figure 1.

Now, if we return to the reference frame connected with the ground (or, by other words, we just stop) we have to add the velocity V to any point of the wheel according to the velocity composition rule (Figure 1 , center). After that it is easy to see that the velocity of the upper point will be 2V with respect to the ground, the center point will have velocity V (as we expected) and the lower point will have zero velocity. As the wheel is rolling, the lowest point does not move with respect to the ground!

It is convenient to think about the rolling as of instantaneous rotation n around the lowest point of the rolling object with the angular velocity $\omega = V/R$ (Figure 2).

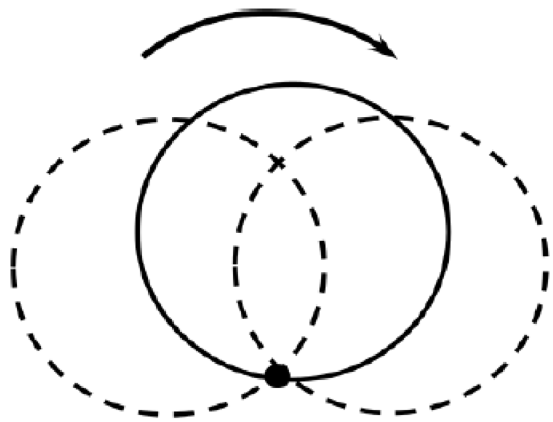


Figure 2.

In a split second another point becomes the lowest one and stops with respect to the ground. This picture gives a recipe how to find the velocity of any point of the rolling disk with respect to the ground. It is illustrated by Figure 3.

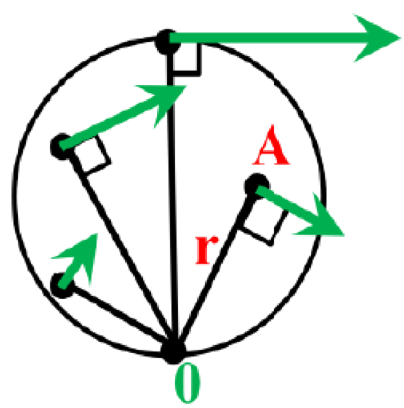
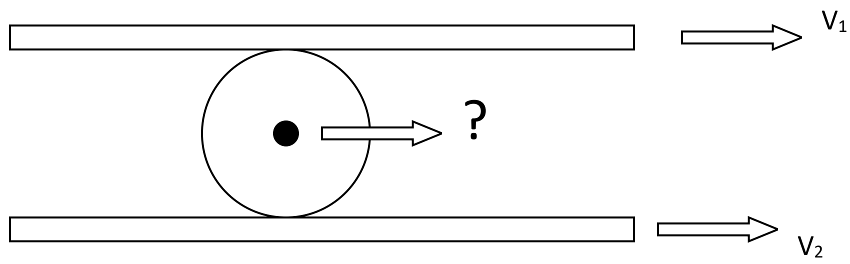


Figure 3.

If you are going to find the instantaneous velocity of a certain point A of the rolling disc, you have to draw a line connecting this point with the lowest point of the disc. The velocity of the point will be directed perpendicularly to this line. The magnitude of the velocity $V_A = \omega r$, where r is the distance between point A and the lowest point of the disc.

Problem:

A cylinder is placed between two plates which move in the same direction with speeds V_1 and V_2. Find angular velocity of the cylinder's rotation and the speed of the cylinder's axes with respect to the ground (see Figure below).


V_1
?
V_2