Homework 5.

Tangential and angular acceleration.

We have been discussing uniform circular motion. This type of motion is characterized by a constant angular velocity ω. It is accelerated because the linear velocity v changes its direction.

Let us consider the situation when a driver presses the gas pedal and the wheels of the car are rotating faster and faster. As the magnitude of the angular velocity increases, the object revolves faster and faster, which means that its linear velocity increases as well. In this case we have a component of the acceleration which is directed along the velocity, i.e. along the tangent line to the trajectory. This acceleration we will call ***tangential*** acceleration.

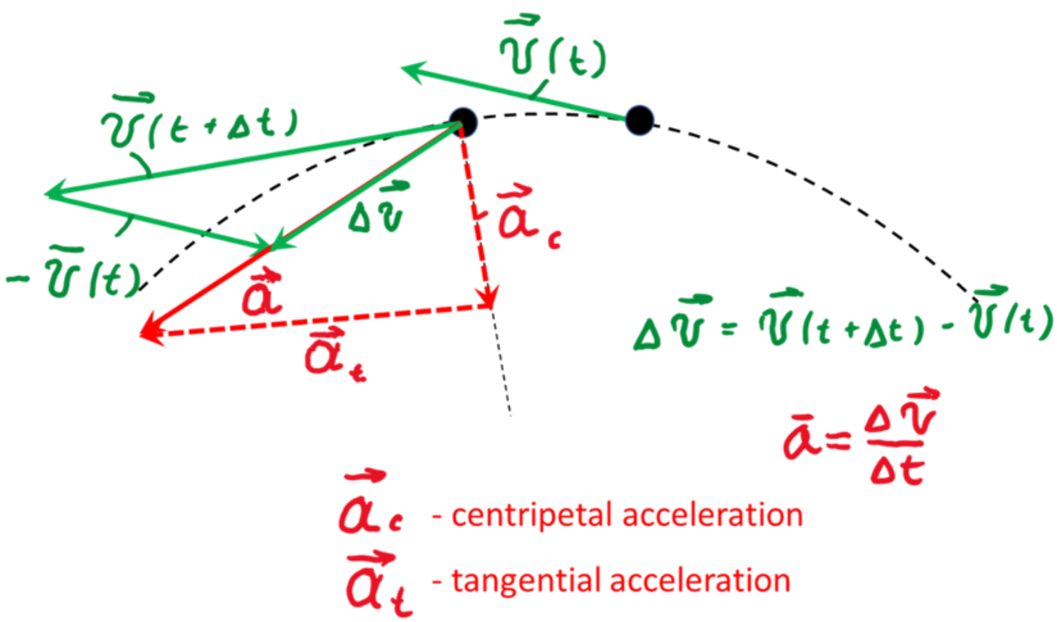

Figure 1. Centripetal and tangential acceleration.

In this case the angular velocity is changing as well and we are dealing with the ***angular acceleration*** $\boldsymbol{\beta}$. The magnitude of the angular acceleration can be calculated as

$$\beta = \frac{a_t}{R} \qquad (1)$$

Here $\boldsymbol{a_t}$ is the magnitude of the tangential acceleration, and $\boldsymbol{R}$ is the radius of the circular trajectory. Since angular velocity is a vector (directed along the rotation axis, just to remind) the angular acceleration takes place any time the angular velocity changes its magnitude and/or direction (for example you are trying to turn the axis of a rotating top). We will consider the situation when just the magnitude of angular velocity is changing. We will also assume that the angular acceleration $\boldsymbol{\beta}$ does not change, so it is a uniformly accelerated motion.

In rotational kinematics $\boldsymbol{\beta}$ is assumed to be known and a typical problem is to compute the position (in this case it is the total turning angle) of an object at any moment of time if the initial angle and initial angular velocity are given.

To better understand rotational kinematics let us go back to the uniformly accelerated rectilinear motion. The formulae below describe the linear displacement and velocity as functions of time:

$$\vec{D} = \overrightarrow{V_0}t + \frac{\vec{a}t^2}{2} \quad (2)$$

$$\vec{V} = \overrightarrow{V_0} + \vec{a}t \quad (3)$$

Here $\boldsymbol{D}$ is the displacement, $\boldsymbol{V_0}$ is the initial velocity, $\boldsymbol{V}$ is the velocity after time $\boldsymbol{t}$ and $\boldsymbol{a}$ is the acceleration. The initial position of the object is taken as a "zero displacement" point.

Similarly, we can introduce the formula for the turning angle – "angular displacement" $\boldsymbol{\alpha}$:.

$$\vec{\alpha} = \overrightarrow{\omega_0}t + \frac{\vec{\beta}t^2}{2} \quad (4)$$

$$\vec{\omega} = \overrightarrow{\omega_0} + \vec{\beta}t \quad (5)$$

Like we did in linear kinematics, we can remove arrows, but ascribe proper signs for all vectors. We can choose a "positive direction" – clockwise or counterclockwise. From now on, any turn along the positive direction is positive, any turn against – negative.

Important!! Angular acceleration should not be confused with centripetal acceleration. The latter is nonzero even in the case of a uniform circular motion. Angular acceleration is nonzero only if the angular velocity is changing.

Problems:

1. You are swinging a sling in a horizontal plane with the angular acceleration of 2 rad/s^2. The initial angular velocity is 0. Find the centripetal acceleration of the sling in 10seconds if the length of the sling is 50cm.

2. A car's driver presses the brake pedal of the car moving at 50 km/h. The car passes 20m in 2 seconds and stops. Find angular acceleration of the wheels if their radius is 50 cm.