THE UNIVERSAL LAW OF GRAVITY

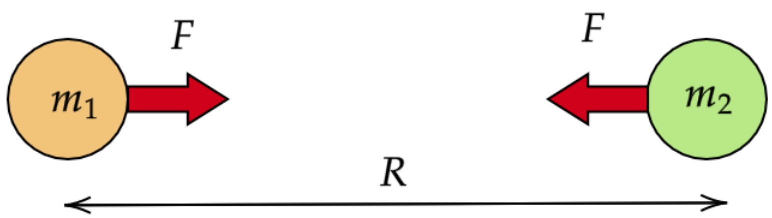
DECEMBER 18, 2023

Theory recap

We have already discussed that any object near the Earth is attracted to the Earth with the force mg where m is object's mass and g is the free fall acceleration. But this formula could only work near the surface of the Earth as the strength of gravity should decrease as one gets further away from the Earth. How does gravity change with distance? Furthermore, we have discussed that different planets have different values of free fall acceleration. Which properties of a planet determine the free fall acceleration on its surface? Finally, is gravitational attraction somehow restricted to cosmic objects or does it act between any objects, no matter how small they are?

The answer to all these questions about gravitational force is contained in the Newton's universal law of gravity. Newton has discovered, that the force of gravity acts between any two objects and depends only on the objects masses and the distance between them. The formula for the gravitational force between two (small) objects of masses m_1 and m_2 which are at the distance R from each other, is

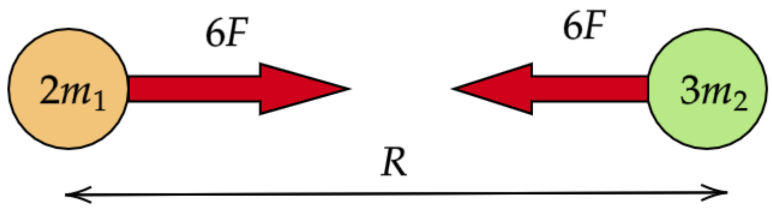
$$\boxed{F = \frac{Gm_1m_2}{R^2}} \tag{1}$$



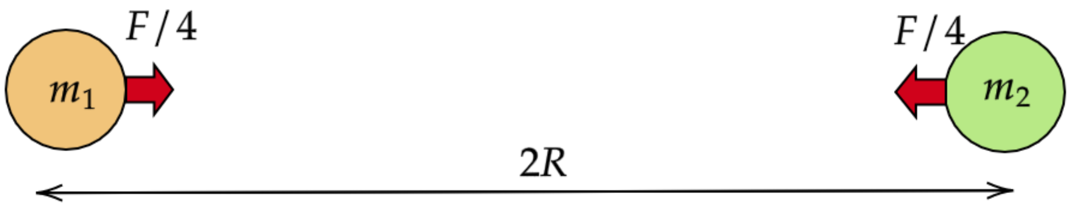
G in this formula is called **gravitational constant** and it is one of the fundamental constants of nature. It sounds amazing, but this formula describes the gravitational force between any two objects in the universe: it could tell us how stars attract each other, or how the Earth pulls the Moon, or how the Earth pulls an ant, or how an ant pulls another ant! Any two objects experience gravitational attraction and G is absolutely the same for any of them. The numerical value of G is

$$G = 6.67 \cdot 10^{-11} \frac{\mathrm{N} \cdot \mathrm{m}^2}{\mathrm{kg}^2}$$

Note that the formula (1) for gravitational force includes a product of masses, $m_1 \cdot m_2$. Therefore, if we double mass of the first object and triple the mass of the second object, the gravitational force between them will increase $2 \cdot 3 = 6$ times (see figure below).

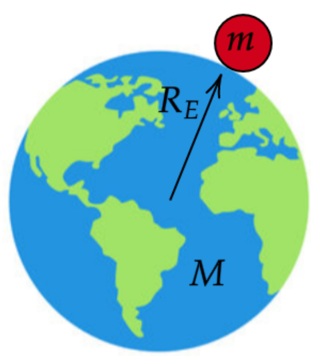


Gravitational force quickly decreases with distance between the objects. Distance squared in the denominator of formula (1) tells us that if we put two objects two times father, their gravitational attraction will become four times weaker (see figure below).



Force is a vector quantity, so we need to specify the direction of gravitational force. Direction of gravitational force is such that the objects attract each other. It means that the gravitational force between two objects pulls these two objects in opposite directions (towards each other) in agreement with Newton's third law: action and reaction are equal and opposite.

Finally, what does the universal law of gravity (1) tell us about the free fall acceleration? Consider a small object of mass m near the surface of the Earth. Let us call the Earth's radius R_E and the Earth's mass M.



On one hand, we know that the force of gravitational attraction between the object m and the Earth is mg. On the other hand, we can use our formula (1) to calculate the same force in a different way. Let us note, that formula (1) as it stands is not directly applicable here: in formula (1) we assumed that the distance between the objects is much larger than any of their sizes which is not the case here. However, it can be proven (which we do not do here) that the same formula is valid for a spherically symmetric object (and the Earth is spherically symmetric with a good accuracy) of any size. In that case we just need to use as R the distance between the small object m and the center of the Earth. If m is on the surface of the Earth, this distance is equal to the radius of the Earth R_E. Therefore, the force of attraction of the object m to the Earth is

$$F = \frac{GMm}{R_E^2} = mg$$

We have also explicitly written that the same force is usually expressed as mg. By canceling the common factor of m between the two expressions, we arrive to the expression for the free fall acceleration g via mass and radius of the Earth:

$$g = \frac{GM}{R_E^2}$$

The same formula would work for any planet: we see that free fall acceleration grows with mass of the planet but decreases with the size of the planet.

Homework

1. Find the gravitational force between two humans of mass 70 kg if the distance between them is 10 m.

2. Imagine that you are a NASA engineer in the early days of a Mars exploration program and you need to come up with a project of the first Martian rover. For this project it is critical to know the free fall acceleration on Mars, but no one has been there yet to measure it. Luckily, from astronomical measurements you know mass and radius of Mars. Mass of Mars is $M_M = 6.4 \cdot 10^{23}$ kg and radius of Mars is $R_M = 3400$ km. Use this information to calculate the free fall acceleration on Mars for success of the rover project.

***3.** Find the gravitational force acting on you from the Earth, the Moon and the Sun. Rank them from the strongest to the weakest. Use Google search to find the necessary masses and distances (try to use a credible source).

***4.** Find the mass of the Earth knowing its' radius $R_E \approx 6400$ km.