Classwork/Test

Due at the end of the class.

Main Algebraic Identities/formula

$$a^{-n} = \frac{1}{a^n}$$

$$(a^m)^n = a^{mn}$$

$$\frac{m}{n} = \sqrt[n]{a^m}$$

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a-b)(a+b)$$

Arithmetic series

$$a_n = a_1 + (n-1)d$$

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

$$d = \frac{a_s - a_t}{s - t}$$

$$S = \frac{(a_1 + a_n) \times n}{2}$$

Geometric series

$$a_n = a_1 \times q^{n-1}$$

$$a_n = \sqrt{a_{n-1} . a_{n+1}}$$

$$S_n = a_1 \times \frac{(1 - q^n)}{1 - q}$$

$$\mathrm{S} = \frac{a_1}{1 - q}$$

Binomial coefficients

$nC_k = \binom{n}{k}$ = the number of paths on the chessboard going k units up and n − k to the right
= the number of words that can be written using k ones and n − k zeroes
= the number of ways to choose k items out of n (***order doesn't matter***)

- Formula for binomial coefficients

There is an explicit formula to calculate $\binom{n}{k}$:

$$\binom{n}{k} = \frac{n(n-1)\dots(n-k+1)}{k!} = \frac{n!}{(n-k)!\,k!}$$

- Formula for permutations (the number of ways of choosing k items out of n when ***the order matters***): Compare it with the number of ways of choosing k items out of n when the order matters:

$$_nP_k = n(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!}$$

Binomial probabilities

The binomial coefficients are also useful in calculating probabilities. Imagine that we have some event that happens with probability p ("success") and does not happen with probability $q = 1 - p$ ("failure"). Then the probability of getting k successes in n trials is:

$$P(k\ successes\ in\ n\ trials) = \binom{n}{k} p^k q^{n-k}$$

Where,
- p — probability of success in one try;
- q = 1 – p — probability of failure in one try;
- n — number of trials;
- k — number of successes;
- n – k — number of failures.

Problems:

1. Expand as sums of powers of x $(hint: you\ may\ use\ binomial\ formula)$:
   a. $(2x+5)^2$
   b. $(a+b)^3$
   c. $(1-x)^5$

2. Factor (i.e., write as a product) the following expressions:
   a. $(x-2)^2 - (y+3)^2$
   b. $256 - a^8 b^8$
   c. $x^4 + 4$
   d. $p^4 + 4z^{4n}$
   e. $t^2 - 3/2\ t + 1/2$
   f. $6a^2 - 25a + 24$

3. Simplify:
   a. $\frac{x}{(x^2-y^2)} - \frac{y}{(x+y)^2}$
   b. $\frac{a+b}{(b-c)(c-a)} + \frac{b+c}{(c-a)(a-b)} + \frac{c+a}{(a-b)(b-c)}$

4. $a_5 = 27$ and $a_{27} = 60$. Find the first term a_1 and the common difference d.

5. Find the common difference d in an arithmetic sequence if the 9-th term is 18 and the 11-th term is 44.

6. An arithmetic progression has first term $a_1 = a$ and common difference $d = -1$. The sum of the first n terms is equal to the sum of the first $3n$ terms. Express a in terms of n.

7. Write the first 5 terms of a geometric progression if $a_1 = -20$ and $q = \frac{1}{2}$

8. Calculate the sum: $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^{10}}$

9. A geometric progression has 99 terms, the first term is 12 and the last term is 48. What is the 50-th term?

10. How many ways are there to seat 5 students in a class that has 5 desks? if there are 10 desks?

11. If we toss a coin 10 times, what is the probability that all will be heads? that there will be exactly one tails? 2 tails? exactly 5 tails?

12. You roll a die 100 times. What is the probability of getting a 6 exactly 20 times?

13. A (blindfolded) marksman finds that on the average he hits the target 4 times out of 5. If he fires 4 shots, what is the probability of
    (a) more than 2 hits?
    (b) at least 3 misses