Homework 2: Algebraic operations part2

HW2 is Due October 8

This week we reviewed various types of problems encountered in factorization and simplification of algebraic identities.

Some basic algebraic rules for you to remember (similar to last homework set):

1. **Exponents Laws**

   If a is a real number, n is a positive integer

   $$a^n = a \times a \times \cdots \times a$$

   $$\underbrace{\qquad\qquad}_{\text{n-times}}$$

   n-times

   $$a^0 = 1$$

   $$a^m \times a^n = a^{m+n}$$

   $$a^m \div a^n = a^{m-n}$$

   $$(ab)^n = a^n b^n$$

   $$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

   $$a^{-n} = \frac{1}{a^n}$$

   $$(a^m)^n = a^{mn}$$

2. **Radicals**

   $$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$
   $$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

3. **Main Algebraic Identities**

   $$(a+b)^2 = a^2 + 2ab + b^2$$

   $$(a-b)^2 = a^2 - 2ab + b^2$$

   $$a^2 - b^2 = (a-b)(a+b)$$

   Homework problems on next page

Instructions: Please always write solutions on a ***separate sheet of paper***. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and

some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

1. Simplify

   a. $\sqrt{\dfrac{56}{13}} \cdot \sqrt{\dfrac{26}{7}} =$

   b. $\sqrt{48} =$

   c. $\dfrac{\sqrt{48}}{\sqrt{15}} =$

2. Express the following expressions in the form $2^r 3^s a^m b^n$:
   a. $8a^3 b^2 (27a^3)(2^5 ab) =$
   b. $3^2 (2ab)^3 (16a^2 b^5)(24b^2 a) =$
   c. $16a^2 b^3 (6ab^4)(ab^2)^3 =$

3. Expand as sums of powers of x:
   a. $(2x + 5)^2 =$
   b. $(2 - 4x)^2 =$
   c. $(1 - 2x)^2 =$

4. Factor (i.e., write as a product) the following expressions:

   a. $4x^2 + 8xy + 4y^2$
   b. $9x^2 - 25$
   c. $(x - 2)^2 - (y + 3)^2$
   d. $256 - a^8 b^8$

   e. $(x - 2)^2 - 10(x - 2) + 25$
   f. $3x^3 - x^2 y + 6x^2 y - 2xy^2 + 3xy^2 - y^3$

   g. $a^2 - b^2 - 10b - 25$
   h. $x^4 + 4$
   i. $p^4 + 4z^{4n}$
   j. $a^2 + 3a + 2$
   k. $m^2 - m - 12$
   l. $m^2 - 5/6\, m + 1/6$
   m. $t^2 - 3/2\, t + 1/2$
   n. $6\,a^2 - 25\,a + 24$
   o. $7\,x^2 + 48\,x - 7$

5. Solve the following equations :
   a. $5(x + 1) = 3x + 2$
   b. $(x^2 - 1)(x + 2) = 0$
   c. $\dfrac{x+2}{x+3} = 2$
   d. $(x - 3)(x + 4) = 0$
   e. $x^2 + 4x = 0$
   f. $x^3 + 4x = 0$

6. Prove:
   a. $(a + b)^3 = a^3 + 3a^2 b + 3ab^2 + b^3$
   b. $(a - b)^3 = a^3 - 3a^2 b + 3ab^2 - b^3$
   c. $Find\,(a + b)^4, (a - b)^4\ using\ the\ previous\ results$

7. Amanda has an average of 92 on her seven tests. What should she get on her 8*th* test to have an average of 93?