## REVIEW + CLASSWORK

SEPT. 17, 2023

We are starting this semester with a review of some topics we learned last year.

## Pythagora's Theorem

In a right triangle with legs $a$ and $b$, and hypotenuse $c$, the square of the hypotenuse is the sum of squares of each leg. $c^{2}=a^{2}+b^{2}$. The converse is also true, if the three sides of a triangle satisfy $a^{2}+b^{2}=c^{2}$, then the triangle is a right triangle. Some Pythagorean triples are: $(3,4,5),(5,12,13),(7,24,25),(8.15,17),(9,40,41),(11,60,61),(20,21,29)$.
To generate such Pythagorean triples, choose two positive integers a and $\mathrm{b},(a>b)$. Then plug the values into the sides as shown below:


Can you explain why this method works?
Problem: Given 2 concentric circles, chord AB is 8 cm long and tangent to the smaller of two concentric circles. A and B are points on the larger circle. What is the area between the 2 circles?

Problem: (Mathcounts) Find AB.


Problem: (Mathcounts) Find AG.


The 30-60-90 Triangle


In a right triangle, if one of the angles is given as $30^{\circ}$ or $60^{\circ}$ then this triangle is called 30-60-90 triangle and you know the ratio of the sides. We showed in class that if the smaller leg is $a$ then the hypotenuse is $2 a$ and using Pythagora's theorem one can find the altitude to be $a \sqrt{3}$.

The 45-45-90 Triangle


Given that an angle of a right triangle is $45^{\circ}$, you can compute the other angle and it will also be $45^{\circ}$. This triangle is half a square, when the square is folded along its diagonal. You can use Pythagora's theorem to calculate the hypotenuse.

Problem: What is the area of a regular hexagon whose side is 2 cm ?

Problem: What is the height of three congruent stacked circles with a radius of 12 cm ?


Distance between Two Points


The distance between points $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ can be calculated using Pythagora's theorem in the given right triangle. The hypotenuse is $d$ and we have $d^{2}=\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}$ and $d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$

## Midpoint of a Line Segment

The midpoint of a segment with endpoints $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ is the point with coordinates $\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)$

Exercise: Find the distance between $(5,-4)$ and $(-3,2)$ on the coordinate plane. What is the midpoint?

## Miscellaneous

$\frac{1}{1+\frac{2}{3+\frac{1}{6+12}}}=$
What is the sum of the first 100 positive odd integers?
Find the sum of $\frac{2}{5}+\frac{4}{25}+\frac{8}{125}+\ldots$
Factorize:
(a) $100 x^{8} y^{2}-16 x^{4} y^{6}$ (b) $4 x^{2}-4 x+1$ (c) $a^{2}-2 a+1$ (d) $a^{4}-b^{4}$

Simplify:
(a) $\frac{1}{1+x}-\frac{1}{x-1}$
(b) $\frac{(1+1 / x)}{x+1}$
(c) $\frac{1+1 / x}{1-1 / x}$

