MATH 8 [09/17/2023]
HANDOUT 1: COMBINATORICS REVIEW

Welcome to the new semester at SchoolNova!!

This Fall, we plan to study the following topics:

- Review of combinatorics. Binomial formula.
- Formal logic and proofs. Logic circuits.

In Spring, we will study

- Euclid geometry: axioms, triangles, quadrilaterals and circles
- Number theory: divisibility, factorization, and modular arithmetics.

I ask that each student bring a notebook (preferably quad ruled), pencils and a folder or binder to keep old assignments — you will need them!

As usual, all HW assignments and other information will be posted online at http://www.schoolnova.org. We will try to do much of the homework in class so that you do not need to spend too much time on it at home. **However, you should turn in complete homework, including the work done in class**.

This year we will start learning to appreciate the true rigor of Mathematics. It is enjoyable to be right, and in Math we prove a point with elegant logic instead of "winning an argument" with a dramatic opinion. If in a homework I ask you to prove a statement, please try to make it neat and tidy.

We also plan to participate in two math competitions: Math Kangaroo and American Math Contests (AMC). Math Kangaroo is an international math competition for all ages; you can find more information on their web site at http://www.mathkangaroo.org. The contest is in March. Details of the registration will follow.

AMC (http://amc.maa.org/) is the "official" American Math Olympiad: it is the first level of the competition that eventually leads to the selection of US team for International Math Olympiad. AMC 8 is intended for students in grades 8 and below. You do not have to register individually – just let me know if you are interested.

If you have any questions, please contact me by email: syritsyn@schoolnova.org.

Main Formulas of Combinatorics

- The number of ways to order k items is

$$k! = k(k-1)\dots 2\cdot 1$$

- The number of ways to choose k items out of n **if the order matters**:

$${}_nP_k = n(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!}$$

- The number of ways to choose k items out of n **if the order does not matter**:

$$\binom{n}{k} = {}_nC_k = \frac{n(n-1)\dots(n-k+1)}{k(k-1)\dots 1} = \frac{n!}{(n-k)!k!}$$

These numbers are the ones that appear in Pascal triangle and in many other problems:

$\binom{n}{k} = {}_nC_k =$ The number of paths on the chessboard going k units up and $n-k$ to the right

$=$ The number of words that can be written using k zeros and $n-k$ ones

$=$ The number of ways to choose k items out of n if the order **does not matter**

Combinatorics review problems

Note: you don't always need to compute the actual numbers, because they can be HUGE! For example, in problem 8, it's better to keep $60!$ ("sixty-factorial") in your solution as-is, because the actual number is larger than 10^{80}.

1. A club consisting of 25 people need to choose the president, vice-president, and treasurer. In how many ways can they do this?

2. In a meeting of 25 people, every one of them shakes hands once with every other. How many handshakes was it altogether?

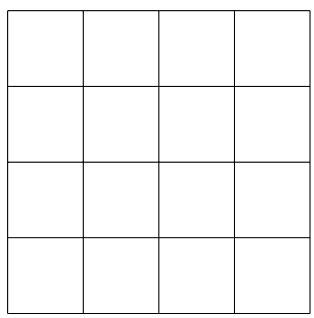
3. There is a round table seating 8. How many ways there are for 8 people to choose their seats at the table? What if we do not distinguish between two seatings which only differ by rotating the table?

4. How many words one can get by permuting letters of the word "tiger"? of the word "rabbit"? of the word "common"? of the word "Mississippi"?

5. If we draw 3 cards out of the deck of 52 cards (4 suits $\times$ 13 values), what are the chances that

- They will all be all spades
- They will be all aces
- That they will be ace of spades, queen of spades, and king of spades, in this order
- That they will be queen of spades, ace of spades, and king of spades, in this order
- * That they will be ace, queen, and king of spades, in some order

6. How many different paths are there on 4×4 chessboard connecting the lower left corner with the upper right corner? What about 5×5? The path should always be going to the right or up, never to the left or down.



7. How many "words" of length 5 one can write using only letters U and R, namely 3 Us and 2 Rs? What if you have 5 Us and 3 Rs? [Hint: it is related to the previous problem – each such "word" can describe a path on the chessboard, U for up and R for right...]

8. A drunkard is walking along a road from the pub to his house, which is located 1 mile north of the pub. Every step he makes can be either to the north, taking him closer to home, or to the south, back to the pub – and it is completely random: every step with can be north of south, with equal chances. What is the probability that after 60 steps, he will end up

(a) at the starting position
(b) 2 steps north from the starting position
(c) 1 steps north from the starting position
(d) 10 steps north from the starting position
(e) 8 steps north from the starting position

***9.** You have 10 books which you want to put on 2 bookshelves. How many ways are there to do it (order on each bookshelf matters)?