MATH 7: HANDOUT 4
ALGEBRAIC EXPRESSIONS AND IDENTITIES

MAIN ALGEBRAIC IDENTITIES

Here is a list of the main algebraic identities we discussed:

1. $(ab)^n = a^n b^n$
2. $\sqrt{ab} = \sqrt{a}\sqrt{b}$
3. $(a+b)^2 = a^2 + 2ab + b^2$
4. $(a-b)^2 = a^2 - 2ab + b^2$
5. $a^2 - b^2 = (a-b)(a+b)$

Replacing in the last equality a by $\sqrt{a}$, b by $\sqrt{b}$, we get

$$(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = a - b$$

which is very helpful in simplifying expressions with roots, for example:

$$\frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2} - 1$$

We also talked about the formulas for the third power (cube) of the sum and difference:

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a-b)^3 = a^2 - 3a^2b + 3ab^2 - b^3$$

There are also formulas for a difference of two cubes and for a sum of two cubes. Notice that we did not have a formula for the sum of two squares!!!

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

We also discussed solving simple equations: linear equation (i.e., equation of the form $ax+b=0$, with a, b some numbers, and x the unknown) and equation where the left hand side is factored as product of linear factors, such as $(x-2)(x+3) = 0$.

Homework problems on back

HOMEWORK

1. Simplify

(a) $\dfrac{42^2}{6^2} =$

(b) $\dfrac{6^3 \times 6^4}{2^3 \times 3^4} =$

(c) $(2^{-3} \times 2^7)^2 =$

(d) $\dfrac{3^2 \times 6^{-3}}{10^{-3} \times 5^2} =$

2. Simplify

(a) $\dfrac{a}{2} + \dfrac{b}{4} =$

(b) $\dfrac{1}{a} + \dfrac{1}{b} =$

(c) $\dfrac{3}{x} + \dfrac{5}{xy} + \dfrac{5}{3a} =$

3. Using algebraic identities calculate

(a) $299^2 + 598 + 1 =$

(b) $199^2 =$

(c) $51^2 - 102 + 1 =$

4. Expand

(a) $(4a - b)^2 =$

(b) $(a + 9)(a - 9) =$

(c) $(3a - 2b)^2 =$

5. Factor

(a) $ab + ac =$

(b) $3a(a + 1) + 2(a + 1) =$

(c) $36a^2 - 49 =$

(d) $a^9 - 27$

6. Find expansions of $(a + b)^4$, $(a - b)^4$ using the previous results.

7. Write each of the following expressions in the form $a + b\sqrt{3}$, with rational a, b:

(a) $(1 + \sqrt{3})^2$

(b) $(1 + \sqrt{3})^3$

(c) $\dfrac{1}{1 - 2\sqrt{3}}$

(d) $\dfrac{1 + \sqrt{3}}{1 - \sqrt{3}}$

(e) $\dfrac{1 + 2\sqrt{3}}{\sqrt{3}}$