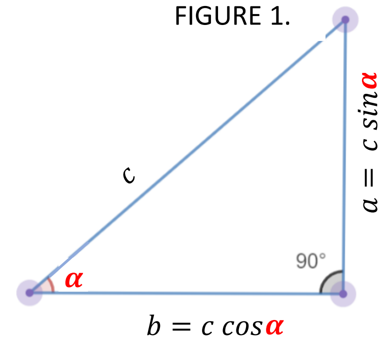
MATH 7

Trigonometry 2, Law of sines.

1. Definition for sin and cos of an angle

In general, for a right-angle triangle, the ***sin*** α and ***cos*** α of the angle are defined as:

FIGURE 1.

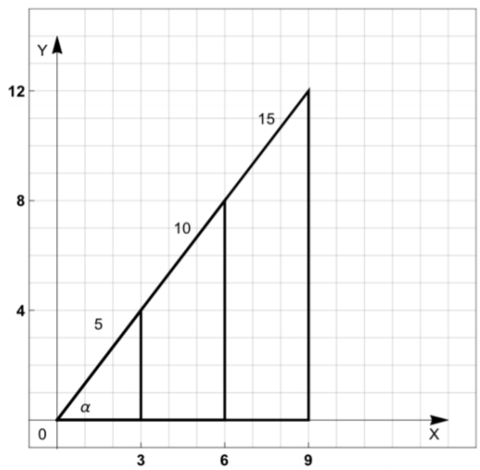


$$\sin(\alpha) = \frac{\text{opposite side}}{\text{hypothenuse}} = \frac{a}{c}, \quad \cos(\alpha) = \frac{\text{adjacent side}}{\text{hypothenuse}} = \frac{b}{c}$$

2. Definition of tangent of an angle

Now we can also define the 3rd trigonometric ratio:

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} = \frac{\text{opposite side/hypothenuse}}{\text{adjacent side/hypothenuse}} = \frac{a}{b}$$



Example: Consider the angle a in the following triangles:

$$\sin(\alpha) = \frac{\text{opposite side}}{\text{hypothenuse}} = \frac{4}{5} = \frac{8}{10} = \frac{12}{15}$$

$$\cos(\alpha) = \frac{\text{adjacent side}}{\text{hypothenuse}} = \frac{3}{5} = \frac{6}{10} = \frac{9}{15}$$

$$\tan(\alpha) = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{4}{3} = \frac{8}{6} = \frac{12}{9}$$

3. Table with values for trigonometric functions

Function	Notation	Definition	0^0	30^0	45^0	60^0	90^0
sine	sin(α)	$\frac{\text{opposite side}}{\text{hypothenuse}}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cosine	cos(α)	$\frac{\text{adjacent side}}{\text{hypothenuse}}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	tan(α)	$\frac{\text{opposite side}}{\text{adjacent side}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

4. Trigonometric identities and the law of sines

- The most prominent trigonometric identity: $\mathbf{\sin^2 \alpha + \cos^2 \alpha = 1}$

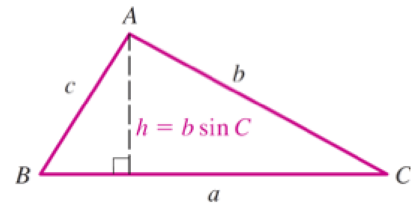
Proof: Pythagoras theorem for Fig1 gives $a^2 + b^2 = c^2$; $(c\sin(\alpha))^2 + (c\cos(\alpha))^2 = c^2$; then divide both sides by c^2 to obtain the identity.

- The law of sines: Given a triangle ΔABC with sides *a*, *b*, and c, the following is always true:

$$\frac{\boldsymbol{a}}{\boldsymbol{sin(A)}} = \frac{\boldsymbol{b}}{\mathbf{sin}(\boldsymbol{B})} = \frac{\boldsymbol{c}}{\mathbf{sin}(\boldsymbol{C})}$$

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

Proof: To see why the Law of Sines is true, refer to the figure. The height of the triangle $h = b\sin(C)$, and therefore the area is: $S = \frac{1}{2}a \times b\sin(C)$. Similarly, $h = c\sin(A)$ and $S = \frac{1}{2}a \times c\sin(B)$. Constructing a height towards side b, $S = \frac{1}{2}b \times c\sin(A)$. Thus, $bc\sin(A) = ac\sin(B) = ab\sin(C)$. Divide by *abc*, to get the law.



Homework problems

All angles are measured in degrees.

1. If a right triangle ΔABC has sides $AB = 3 \times \sqrt{3}$ and $BC = 9$, and side AC is the hypotenuse, find all 3 angles of the triangle.
2. The area of a right triangle is 36 square meters. The legs of the triangle have the ratio of 2 : 9. Find the hypotenuse of the triangle.
3. In a triangle ΔABC, we have $< A = 40^0$; $< B = 60^0$, and AB = 2 cm. What is the remaining angle and side lengths? (Hint: Use Law of sines)
4. In an isosceles triangle, the angle between the equal sides is equal to 30^0, and the height is 8. Find the sides of the triangle.
5. A right triangle ΔABC is positioned such that A is at the origin, B is in the 1st quadrant (coordinates B*x* > 0 and By > 0) and C is on the positive horizontal axis (C*x* > 0 and Cy = 0). If length of side AB is 1, and AB makes a 35^0 angle with positive x-axis, what are the coordinates of B?
6. Consider a parallelogram ABCD with AB = 10, AD = 4 and $< BAD = 50^0$. Find the length of diagonal AC. (Hint: make ΔACM, where $< M$ is 90^0 and point M is on the same line as CD)
7. A regular heptagon (7 sides) is inscribed into a circle of radius 1.
   a. What is the perimeter of the heptagon?
   b. What is the area of the heptagon?
8. In the trapezoid ABCD, AD = 5 cm, AB = 2 cm, and $< A = < D = 70^0$. Find the length BC and the diagonals. [You can use: sin(70^0) ≈ 0:94; cos(70^0) ≈ 0:34.]