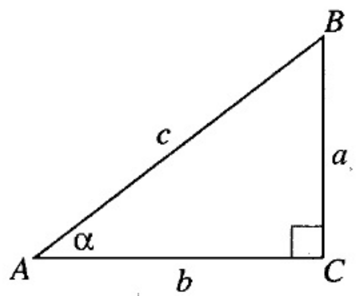
MATH 7
HANDOUT 20: TRIGONOMETRY 1

Trigonometric Ratios

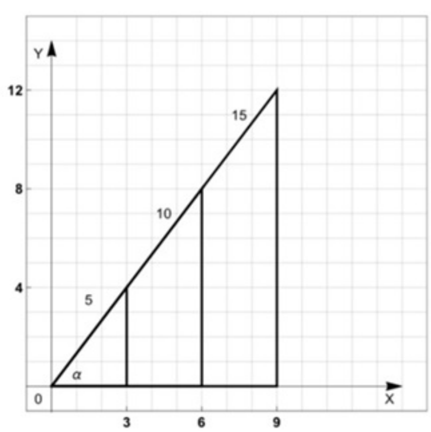


Given a right triangle ABC, we define a function called **sine** as the ratio between the opposite leg and the hypotenuse.

The **cosine** of an angle is the ratio of the adjacent leg to the hypotenuse.

$$\sin\alpha = \frac{opposite}{hypotenuse} = \frac{a}{c}$$
$$\cos\alpha = \frac{adjacent}{hypotenuse} = \frac{b}{c}$$

These ratios are a measure of the angle only, no matter what the size of the sides of this right triangle, the ratio of the sides will be the same.



$$sina = \frac{\text{opposite side}}{\text{hypothenuse}} = \frac{4}{5} = \frac{8}{10} = \frac{12}{15}$$

$$cosa = \frac{\text{adjacent side}}{\text{hypothenuse}} = \frac{3}{5} = \frac{6}{10} = \frac{9}{15}$$

$\sin\alpha = \dfrac{a}{c}$ so $a = c \cdot \sin\alpha$

$\cos\alpha = \dfrac{b}{c}$ so $b = c \cdot \cos\alpha$

Some ratios for the 30-60-90 triangle and the 45-45-90 triangle:

Function	Notation	Definition	0^0	30^0	45^0	60^0	90^0
sine	sin(a)	$\dfrac{\text{opposite side}}{\text{hypothenuse}}$	0	$\dfrac{1}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{\sqrt{3}}{2}$	1
cosine	cos(a)	$\dfrac{\text{adjacent side}}{\text{hypothenuse}}$	1	$\dfrac{\sqrt{3}}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{1}{2}$	0

Homework

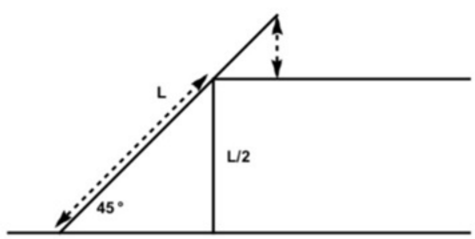
(1) Which one is greater?

  (a) 0 or $\sin 0^{\circ}$
  (b) 1 or $\sin 30^{\circ}$
  (c) $\sin 45^{\circ}$ or $\cos 45^{\circ}$
  (d) $\cos 60^{\circ}$ or $\sin 30^{\circ}$

(2) A tree casts a 60 m long shadow when the angle of elevation of the sun is 30°. How tall is the tree? (Angle of elevation is the angle that the line from the tip of the shadow on the ground to the top of the tree makes with the horizontal).

(3) A ladder of length L is resting on a ledge whose height is half of the ladder's length. The ladder makes a 45° angle with the ground. Express answers in terms of L.

  (a) How long is the portion of the ladder between the ground and the point of contact of the ledge and ladder? (indicated by a long dashed arrow)
  (b) At what height is the top of the ladder above the ledge? (indicated by the short dashed arrow - this forms another right triangle.)



(4) A cruise ship travels north for 3 miles and then north-west for another 3 miles. How far will it end up from its original position (from the start to the end point). (Note: north-west is the direction that bisects the angle between the north and the west).

(5) A ship travels for 3 miles north, then turns and goes for 2 miles northeast, then for another 5 miles north-northeast. Where will it be at the end - how far east and north of the original position? (Northeast means that its direction bisects the angle between the north and east directions, thus forming an angle of 45° with due north. North-northeast means that this direction bisects the angle between north and northeast, thus forming a 22.5° angle with due north).

(6) Consider a regular pentagon inscribed in a circle of radius 1. What is the side length of such a pentagon? (Hint: drop a perpendicular from the center to one of the sides and complete it to form a right triangle).

(7) Consider a parallelogram ABCD with AB=1, AD=3, $\angle A = 40°$. Find the length of diagonals in this parallelogram.

(8) Prove that the area of a triangle ABC can be computed using the formula
$A = \frac{1}{2} \cdot AB \cdot AC \cdot \sin \angle A$ (Hint: what is the altitude from vertex B?)

(9) What is the area of a regular pentagon inscribed in a circle of radius 10? (Make sure to use a trigonometric function.)