MATH 7
HANDOUT 19: VECTORS IN THE PLANE

Vectors

A **vector** is a directed segment. We denote the vector from A to B by $\overrightarrow{AB}$. We will also frequently use lower-case letters for vectors: $\vec{v}$.

We will consider two vectors to be the same if they have the same length and direction; this happens exactly when these two vectors form two opposite sides of a parallelogram. Using this, we can write any vector $\vec{v}$ as a vector with tail at given point A. We will sometimes write $A + \vec{v}$ for the head of such a vector.

Vectors are used in many places. For example, many physical quantities(velocities, forces, etc) are naturally described by vectors.

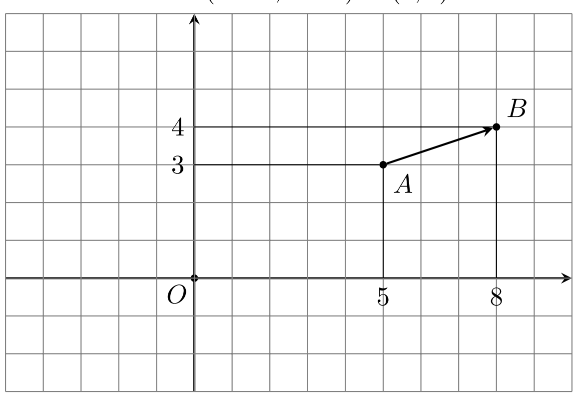
Vectors in coordinates

Recall that every point in the plane can be described by a pair of numbers – its coordinates. Similarly, any vector can be described by two numbers, its x-coordinate and y-coordinate: for a vector $\overrightarrow{AB}$, with tail $A = (x_1, y_1)$ and head $B = (x_2, y_2)$, its coordinates are

$$\overrightarrow{AB} = (x_2 - x_1, y_2 - y_1)$$

For example, on picture below,

$$\overrightarrow{AB} = (8 - 5, 4 - 3) = (3, 1)$$



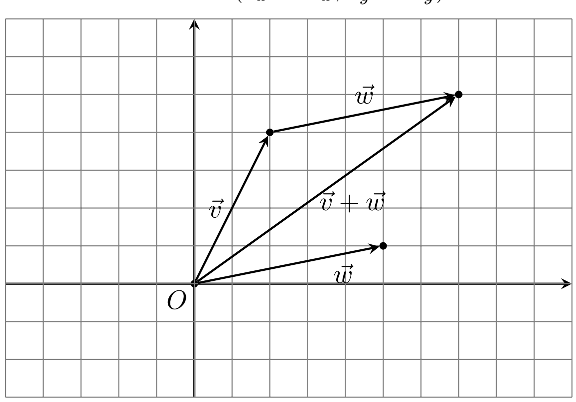
Operations with vectors

Let $\vec{v}$, $\vec{w}$ be two vectors. Then we define a new vector, $\vec{v} + \vec{w}$ as follows: choose A, B, C so that $\vec{v} = \overrightarrow{AB}$, $\vec{w} = \overrightarrow{BC}$; then define

$$\vec{v} + \vec{w} = \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

In coordinates, it looks very simple: if $\vec{v} = (v_x, v_y)$, $\vec{w} = (w_x, w_y)$, then

$$\vec{v} + \vec{w} = (v_x + w_x, v_y + w_y)$$



Theorem. *So defined addition is commutative and associative:*

$$\vec{v} + \vec{w} = \vec{w} + \vec{v}$$
$$(\vec{v_1} + \vec{v_2}) + \vec{v_3} = \vec{v_1} + (\vec{v_2} + \vec{v_3})$$

There is no obvious way of multiplying two vectors; however, one can multiply a vector by a number: if $\vec{v} = (v_x, v_y)$ and t is a real number, then we define

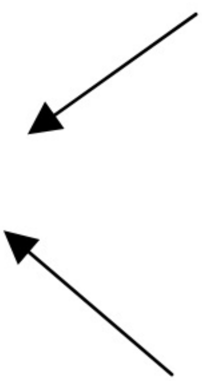
$$t\vec{v} = (tv_x, tv_y)$$

Again, we have the usual distributive properties.

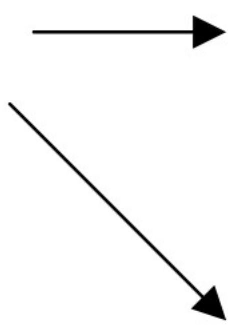
Homework

1. Given $\vec{v} = (3, 4)$ and $\vec{u} = (5, -1)$
  (a) Find $\vec{v} + \vec{u}$
  (b) Find $\vec{v} - \vec{u}$
  (c) Show your result graphically using the planar coordinate system

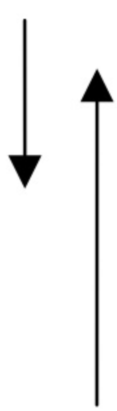
2. Show graphically how to add the vectors below:



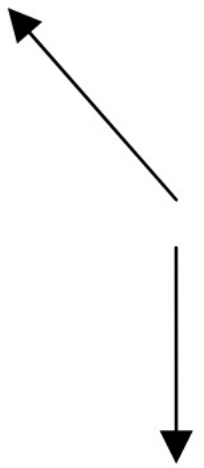
(a)



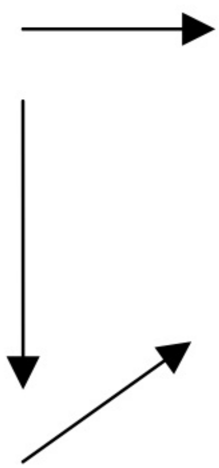
(b)



(c)



(d)



(e)

3. (a) Let $A = (3, 6)$, $B = (5, 2)$. Find the coordinates of the vector $\vec{v} = \overrightarrow{AB}$ and coordinates of the points $A + 2\vec{v}$; $A + \frac{1}{2}\vec{v}$; $A - \vec{v}$.

(b) Repeat part (a) for points $A = (x_1, y_1)$,$B = (x_2, y_2)$

4. Let $A = (x_1, y_1)$,$B = (x_2, y_2)$. Show that the midpoint M of segment AB has coordinates $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$ and that $\overrightarrow{OM} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$.

[Hint: point M is $A + \frac{1}{2}\vec{v}$, where $\vec{v} = \overrightarrow{AB}$].

5. Consider a parallelogram $ABCD$ with vertices $A(0, 0)$, $B(3, 6)$, $D(5, -2)$. Find the coordinates of:

(a) vertex C

(b) midpoint of segment BD

(c) midpoint of segment AC

6. Repeat the previous problem if coordinates of B are (x_1, y_1), and coordinates of D are (x_2, y_2). Use the result to prove that diagonals of a parallelogram bisect each other (i.e., the intersection point is the midpoint of each of them).