MATH 7
HANDOUT 18 COORDINATE GEOMETRY 2

1. Midpoint of a segment

The **midpoint** M of a segment AB with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ has coordinates:

$M(\frac{x_1+x_1}{2}, \frac{y_1+y_2}{2})$

2. Equation of a straight line

The equation of a line is $\mathbf{y = mx + b}$, where m - slope, b - y-intercept.

Given two points $A(x_1, y_1)$ and $B(x_2, y_2)$, the slope can be calculated by dividing $\Delta y = y_2 - y_1$ to $\Delta x = x_2 - x_1$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

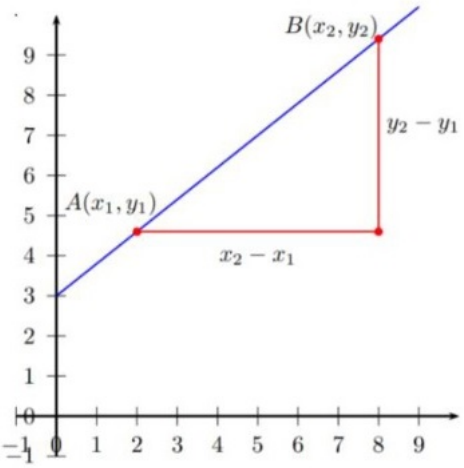
The y-intercept b can be found by plugging in one of the points into $y = mx + b$

Two parallel lines have the same slope $m_1 = m_2$

Two perpendicular lines have the slopes the negative reciprocal of each other. $m_1 \times m_2 = -1$ or $m_2 = -\dfrac{1}{m_1}$

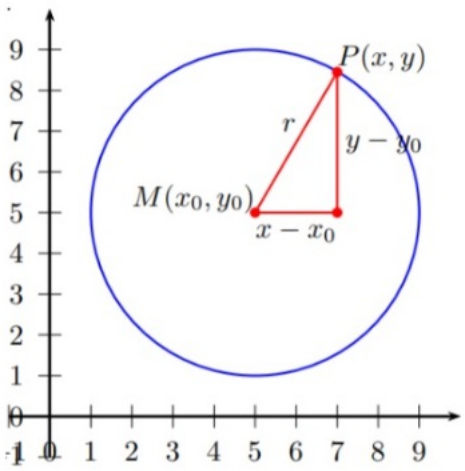
3. Distance between two points

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ can be found by using Pythagora's Theorem in the following right triangle on the graph: $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$, so $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



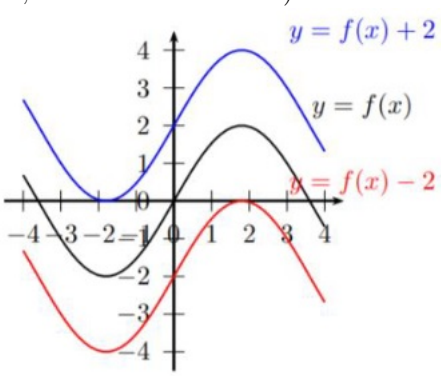
4. Equation of a Circle

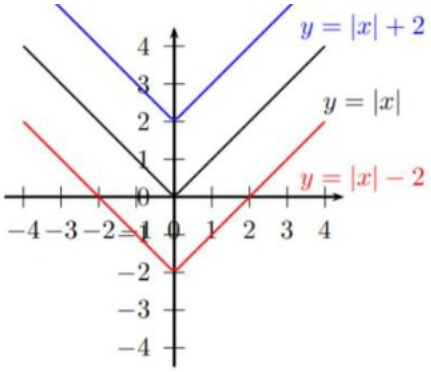
The circle can be defined as all points $P(x, y)$, that are at a distance r from a point $M(x_0, y_0)$ (the center of the circle). We can use the distance between two points to derive the equation of this circle: $r = \sqrt{(x - x_0)^2 + (y - y_0)^2}$. Square it, to get the equation of a circle with center $M(x_0, y_0)$ and radius r: $(x - x_0)^2 + (y - y_0)^2 = r^2$



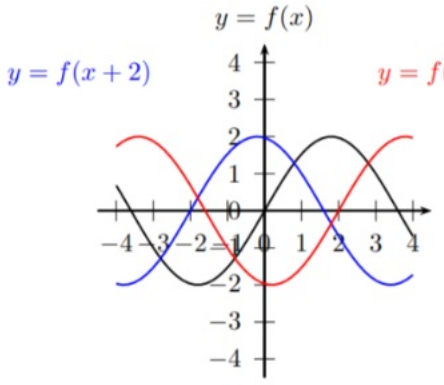
5. Function Transformations

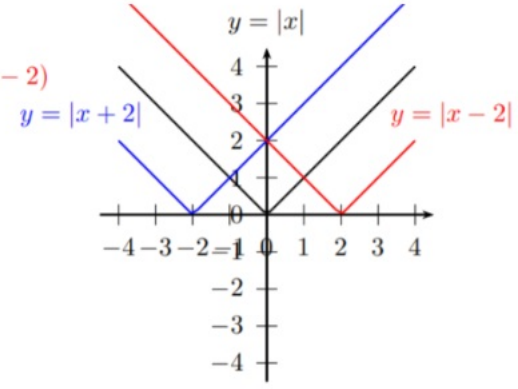
Vertical translations: Adding a constant c to $f(x)$ shifts the graph c units up (if c is positive, if c is negative, it shifts c units down)





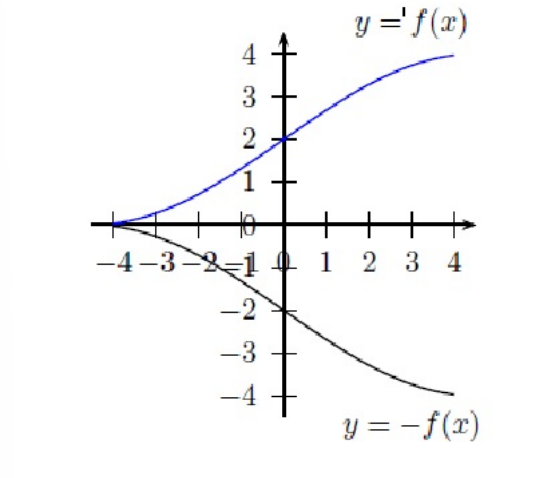
Horizontal translations: Adding a constant c to x, shifts the graph by c units left if c is positive, if c is negative it shifts the graph right by c units.





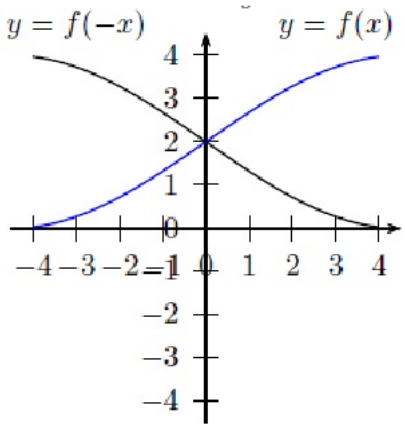
6. Reflection around the x-axis

If $y = f(x)$, then the reflection around the x-axis is $y = -f(x)$:



7. Reflection around the y-axis

Replacing x by $-x$, reflects the graph around the y-axis:



8. Problems

1. Let C be a circle with center (0,1) and radius 2, and l- the line with slope 1 going through the origin. Find the intersection points of the circle C and the line l, and compute the distance between them.

2. There are two lines defined in a plane.

(a) What are the possible orientations of those two lines and how many crossing points could they have? Please explain.

(b) Sketch the graphs of the functions $y = |x + 1|$ and $y = -x + 0.25$. How many crossing points do they have?

(c) How many solutions do you think this equation has?
$|x + 1| = -x + 0.25$
(Note: just answer how many are there. Your are solving graphically - look at your graph. Justify your answer about the number of solutions)

(d) Solve the above equation algebraically. What is the meaning of the solution (x, y)?

3. The distance from a point P=(u, v) to a line expressed in a standard form as $ax + by + c = 0$ is given by the formula: $d = \dfrac{au + bv + c}{\sqrt{a^2 + b^2}}$
(if you wish to challenge yourself you could prove the formula for the case when c=0). Here, we will just use this formula to calculate the distance between a line and a point and then check the value by graphing the line and the point and calculating the distance from the graph.

(a) If the line l_1 defined by $3x + 6y = 0$ and the point P is at $(5, 4)$. Using the formula calculate the distance between P and the line.

(b) Draw a line l_1 defined by $3x + 6y = 0$ (express the equation in slope - intercept form) and the point P=(5,4). Find the equation of the line l_2 which passes through P and is perpendicular to line l_1. Find the point M where the two lines cross (graphically or algebraically). Calculate the distance PM using the coordinates of the two points. Is this value the same as the one you found in (a) ?

4. Sketch the graphs of the following functions. Then, shift each function 2 units up, 2 units right, and reflect with respect to the x-axis. Sketch the result and write the equation.

(a) $y = x$
(b) $y = x^2$
(c) $y = \dfrac{1}{x}$
(d) $y = \sqrt{x}$

5. Use desmos graphing website to draw each graph. Is this what you expected to see? Do not attach, just think about the result.

(a) Draw the graph of the equation $x^2 + y^2 - 1 = 0$
(b) Draw the graph of $xy = 0$
(c) Draw the graph of the equation: $x^2 + y^2 = 0$
(d) Draw the graph of the equation $(x^2 + y^2 - 1)(x^2 + (y - 1)^2 - 1) = 0$
(e) Draw the graph of the equation $(x^2 + y^2 - 1)^2 + (x^2 + (y - 1)^2 - 1)^2 = 0$