

Math 7: Handout 7

Arithmetic Sequences.

Arithmetic Sequences

A sequence of numbers is an **arithmetic sequence** or **arithmetic progression** if the difference between consecutive terms is the same number, the **common difference** or d .

Example: The sequence 1, 5, 9, 13, 17, ... is an arithmetic sequence because the difference between consecutive terms is $d = 4$.

We can also find the n -th term if we know the 1st term and d ?

Example: What is a_{100} in the example above?

$$a_1 = 1$$

$$a_2 = a_1 + d = 1 + 4 = 5$$

$$a_3 = a_2 + d = (a_1 + d) + d = a_1 + 2d = (1 + 4) + 4 = 1 + 2 \times 4 = 9$$

$$a_4 = a_3 + d = (a_2 + d) + d = ((a_1 + d) + d) + d = a_1 + 3d = 1 + 3 \times 4 = 13$$

The pattern is:

$$a_n = a_1 + (n - 1)d$$

$$a_{100} = a_1 + 99d = 1 + 99 \times 4 = 397$$

Properties of an Arithmetic Sequence

A useful property of an arithmetic sequence is that any term is the arithmetic mean of its neighbors:

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

Proof:

$$a_n = a_{n-1} + d$$

$$a_n = a_{n+1} - d$$

Adding these two equalities gives us:

$$2a_n = a_{n-1} + a_{n+1}$$

from where we can get what we need.

Another property of arithmetic sequences is that we can find the common difference d if we know any two terms a_m and a_n :

$$d = \frac{a_m - a_n}{m - n}$$

Sum of an Arithmetic Sequence

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n = n \times \frac{a_1 + a_n}{2}$$

Proof: To prove this, we write the sum in 2 ways, in increasing and decreasing order:

$$S_n = a_1 + a_2 + \cdots + a_n$$

$$S_n = a_n + a_{n-1} + \cdots + a_1$$

Adding these two expressions up and noticing that $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \dots$ we get:

$$2S_n = (a_1 + a_n) \times n$$

$$S_n = n \times \frac{a_1 + a_n}{2}$$

Homework

1. Write the first 5 terms of an arithmetic sequence if $a_1 = 7$ and $d = 2$.
2. What are the first 2 terms for the sequence
 $a_1, a_2, -9, -2, 5, \dots$?
3. $a_{10} = 131$ and $d = 12$. What is a_1 ?
4. $a_5 = 27$ and $a_{27} = 60$. Find the first term a_1 and the common difference d .
5. Find the common difference d in an arithmetic sequence if the 9-th term is 18 and the 11-th term is 44.
6. In the arithmetic progression 5, 17, 29, 41, ... what term has a value of 497?
7. Find the sum of the first 10 terms for the series: 4, 7, 10, 13, ...
8. Find the sum of the first 1000 odd numbers.
9. Find the sum $2 + 4 + \dots + 2018$.
10. In a given arithmetic progression, the first term is 6, and the 87-th term is 178. Find the common difference of this arithmetic progression, and give the value of the first five terms.
11. A triangle has sides of length 3,4,5. What is the inradius r of the circle inscribed in this triangle? (write the area of the triangle in 2 ways. $A=rs$, where s is the semi-perimeter of the triangle).