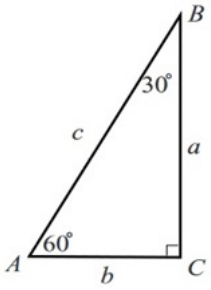
MATH 7: HANDOUT 6

30-60-90 TRIANGLE



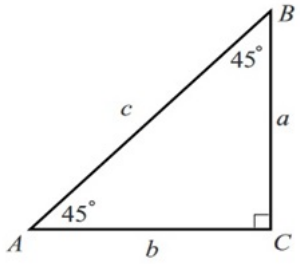
The right triangle that has an angle of $30°$ and another of $60°$ is called a 30-60-90 triangle. You can also think of it as half of an equilateral triangle. The ratio of the sides of the triangle are $1 : \sqrt{3} : 2$. The hypotenuse AB is twice the smallest leg AC. If b=1, then c=2 and we can find out a by using Pythagora's theorem $c^2 = a^2 + b^2 = a^2 + 1^2 = 2^2$, $a^2 = 4 - 1 = 3$, $a = \sqrt{3}$

The ratio of the smallest leg to the hypotenuse is always $\frac{b}{c} = \frac{1}{2}$ and the ratio of the larger leg to the hypotenuse is always $\frac{a}{c} = \frac{\sqrt{3}}{2}$ for this type of triangle. We will revisit these ratios when we learn trigonometry.

Don't try to approximate a square root unless the problem explicitly asks you to. Same goes for π in some geometry problems. Very often the square root or π cancels and if you use decimals, you end up doing more work without getting the exact answer.

In an equilateral triangle of side length a, the altitude is $h = \frac{a\sqrt{3}}{2}$.

45-45-90 TRIANGLE

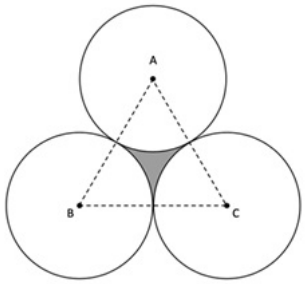


You can think of this right triangle also as being half a square. In this triangle, the base angles are $45°$ both, so this is an isosceles triangle and $a = b$. The ratio of the sides are $1 : 1 : \sqrt{2}$. Let's take $a = b = 1$, then we could use Pythagora's theorem to find side c: $c^2 = a^2 + b^2 = 1^2 + 1^2 = 2$, $c = \sqrt{2}$

In a square of side length s, the length of the diagonal is $s\sqrt{2}$.

HOMEWORK

1. What is the altitude and area of an equilateral triangle of side length 4?
2. What is the area and diagonal length of a square with side length 4?
3. A regular hexagon is inscribed in a circle of radius 6. What is the area of the hexagon?
4. A regular hexagon is inscribed in a circle of radius R. What is the area inside of the circle and outside the hexagon?
5. What is the diagonal of a cube of side length 2?
6. What is the height of a stack of 3 congruent circles of radius 3cm?



(Hint: Consider the equilateral triangle formed by connecting the center of the circles)

7. In the previous problem, what is the shaded area?
8. Rationalize the denominator:

(a) $\dfrac{1}{1+\sqrt{5}}$

(b) $\dfrac{1}{1-2\sqrt{3}}$

(c) $\dfrac{1}{4\sqrt{3}+1}$

(d) $\dfrac{2}{2\sqrt{2}-1}$