MATH 7: HANDOUT 5
RATIONALIZING THE DENOMINATOR

Announcement: Math Kangaroo registration is now open. Please check the instructions on our main webpage www.schoolnova.org.

RATIONALIZING THE DENOMINATOR

To rationalize the denominator means that if there is a radical in the denominator you want to eliminate it. In case of expressions of the form $a+\sqrt{b}$ or $a-\sqrt{b}$ in the denominator, we could multiply the original fraction with the conjugate of this binomial. The conjugate is the same expression but the sign between the terms is changed (from + to - or from - to +).

Let us take a look at an example: $\dfrac{2}{3+\sqrt{3}}$

The conjugate of $3+\sqrt{3}$ is $3-\sqrt{3}$.

$$\frac{2}{3+\sqrt{3}}=\frac{2}{3+\sqrt{3}}\cdot\frac{3-\sqrt{3}}{3-\sqrt{3}}=\frac{2(3-\sqrt{3})}{3^2-(\sqrt{3})^2}=\frac{6-2\sqrt{3}}{(9-3)}=\frac{6-2\sqrt{3}}{6}=\frac{3-\sqrt{3}}{3}$$

In general, the conjugate of $a+\sqrt{b}$ is $a-\sqrt{b}$ and the conjugate of $a-\sqrt{b}$ is $a+\sqrt{b}$.

MAIN ALGEBRAIC IDENTITIES

$$\begin{aligned}(a+b)^2&=a^2+2ab+b^2\\(a-b)^2&=a^2-2ab+b^2\\a^2-b^2&=(a-b)(a+b)\\a^3-b^3&=(a-b)(a^2+ab+b^2)\\a^3+b^3&=(a+b)(a^2-ab+b^2)\end{aligned}$$

The proofs for the last two identities are given as homework problems.

HOMEWORK

1. Rationalize the denominator:

(a) $\dfrac{1}{1+\sqrt{2}}$

(b) $\dfrac{1}{1-\sqrt{3}}$

(c) $\dfrac{1}{1+2\sqrt{3}}$

(d) $\dfrac{1+\sqrt{3}}{1-\sqrt{3}}$

(e) $\dfrac{a+b\sqrt{5}}{\sqrt{5}}$

(f) $\dfrac{\sqrt{p+q}-\sqrt{p-q}}{\sqrt{p+q}+\sqrt{p-q}}$

2. Prove:
$a^3-b^3=(a-b)(a^2+ab+b^2)$
$a^3+b^3=(a+b)(a^2-ab+b^2)$

3. Find the expansions of $(a+b)^3$, $(a-b)^3$.

4. Simplify:

(a) $\sqrt{\dfrac{65}{14}}\cdot\sqrt{\dfrac{13}{11}}\cdot\sqrt{\dfrac{77}{1}}$

(b) $\sqrt{72}$

(c) $\dfrac{\sqrt{72}}{\sqrt{18}}$

5. Expand as sums of powers of x:

(a) $(3x+4)^2 =$

(b) $(5-2x)^2 =$

(c) $(2x-3)^2 =$

6. Factor (i.e., write as a product) the following expressions:

(a) $a^2 - 2a + 1$

(b) $a^2 + 4ab + 4b^2$

(c) $16x^2 - 32xy + 16y^2$

(d) $16x^2 - 25$

(e) $(x-5)^2 - (y-4)^2$

(f) $(x-5)^2 - 12(x-5) + 36$

(g) $a^2 - b^2 - 12b - 36$

(h) $x^4 + 64$ [Hint: add and then subtract $16x^2$.]

(i) $x^2 - 7$ [Hint: $7 = (\sqrt{7})^2$.]

(j) $a^4 - b^4$ [Hint: $a^4 = (a^2)^2$.]

7. Solve the following equations.

(a) $2(x-3) = 4x + 1$

(b) $(x^2+2)(x-1) = 0$

(c) $\frac{x-1}{x+4} = \frac{3}{2}$

(d) $(x-2)(x+5) = 0$

(e) $x^2 - 5x = 0$

(f) $x^3 - 5x = 0$

8. Gilda has a bag of marbles. She gives 20% of them to her friend Pedro. Then Gilda gives 10% of what is left to another friend, Ebony. Finally, Gilda gives 25% of what is now left in the bag to her brother Jimmy. What percentage of her original bag of marbles does Gilda have left for herself?