Math 6: Homework 2.7

Coordinate Geometry

In this section of the course, we are going to study coordinate geometry. The basic notion is the **coordinate plane** – a plane with a given fixed point, called the **origin**, as well as two perpendicular lines – **axes**, called the x**-axis** and the y**-axis.**

The midpoint

The midpoint M of a segment AB with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ has coordinates:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Straight line

This curve is called the **graph** of the given relation.
Every relation (**equation**) of the form:

$$y = mx + b$$

where *m, b* are some numbers, defines a **straight line**. The **slope** of this line is determined by *m*: as you move along the line, y changes *m* times as fast as x, so if you increase x by 1, then y will increase by m. And *b* is a **y-intercept**, it determines where the line intersects the vertical axis (y-axis).

In other words, given two points $A(x_1, y_1)$ and $B(x_2, y_2)$ **slope** can be computed by dividing change of y: $y_2 - y_1$ by the change of x: $x_2 - x_1$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Two non-vertical lines are **parallel** if and only if they have the **same slope**.

Perpendicular lines slope give -1 if multiplied. Given any two lines k and l in the coordinate plane, let their slopes be u and v. If $k \perp l$, then $\boldsymbol{uv} = -\mathbf{1}$. On the other hand, if $uv = -1$, then $k \perp l$.

The distance between two points

$P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by the following formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

This formula is a straightforward consequence of the Pythagoras' Theorem.

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Solve it on a separate piece of paper

1. 3 points $A(0,0)$, $B(1,3)$, $D(5,-2)$ are vertices of a parallelogram $ABCD$. What are the coordinates of point C?

2. In this problem you will find equations that describe some lines.

   a. What is the equation whose graph is the y-axis?

   b. What is the equation of a line whose points all lie 5 units above the x-axis?

   c. Is the graph of $y = x$ a line? Draw it.

   d. Find the equation of a line that contains the points $(1,-1)$, $(2,-2)$, and $(3,-3)$.

3. For each of the equations below, draw the graph, then draw the perpendicular line (going through the point (0, 0)) and then write the equation of the perpendicular line:

   (a) $y = 3x$ (b) $y = -1/2x$

4. Find the equation of the line through $(1,1)$ with slope 2.

5. Find the equation of the line through points $(1,1)$ and $(3,7)$. [Hint: what is the slope?]

6. Find the intersection point of a line $y = x - 3$ and a line $y = -2x + 6$. Sketch the graphs of these lines.

7. a) Find the area of a triangle with vertices at (5,4), (0,3), (-1,-2).

   b) Show that the quadrilateral with the vertices at (-1,-2), (4,-1), (5,4), (0,3) is a rhombus. Then, find its area.

8. Sketch graphs of the following functions:

   (a) $y = |x| + 1$ (b) $y = |x + 1|$ (c) $y = |x - 5| + 1$

9. It is conventional to consider lines parallel to the x-axis as having slope 0, and lines parallel to the y-axis as having slope infinity or undefined. Prove that two sides of a triangle cannot be parallel and explain why this creates a problem in defining the slope of a line parallel to the x or y axes.