Math 6: Homework 2.3
Geometric Sequences Continued

Geometric Sequence (Reminder)

A sequence of numbers is a geometric sequence or geometric progression if the next number in the sequence is the current number times a fixed constant called the common ratio or q.
The sequence 6,12,24,48,... is a geometric sequence because the next number is obtained from the previous by multiplication by q = 2.

$$b_n = b_1 q^{n-1}$$

A Property of Geometric Sequences

A property of a geometric sequence is that any term is the geometric mean of its neighbors. For example, sequence 6,12, 24, 48.. :

$$b_2 = \sqrt{b_1 \cdot b_3} = \sqrt{6 \cdot 24} = 12.$$

$$b_n = \sqrt{b_{n-1} \cdot b_{n+1}}$$

Sum of a Geometric Sequence

Let's try to sum $1 + 2 + 4 + \cdots + 64$. For purposes of working with this sum, let it be called S, i.e. $S = 1 + 2 + 4 + \cdots + 64$. Then I can notice that $2S = 2 + 4 + 8 + \cdots + 128$; subtract the original sum to get $2S - S = 128 - 1$ (everything else cancels out). Thus $S = 127$. What did we do here? We multiplied by 2, which lined up the terms of the sequence to the next term over.
In the geometric sequence 1, 2, …, 64, the common ratio is $q = 2$.

Let's do this in general. Let $a_1, \dots, a_n$ be a geometric sequence with common ratio q.

$$S_n = b_1 + b_2 + b_3 + \cdots + b_n = \frac{b_1(1 - q^n)}{1 - q}$$

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Problems

1. Write the first 5 terms of a geometric progression if $b_1 = -20$ and $q = ½$

2. Calculate: $S = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243}$.

3. After opening his business, John earned 7000$ in the first month. Each next month his income increased by 400% compared to the previous one. How much did he earn in the first 4 months?

4. Calculate: $S = 1 - 2 + 2^2 - 2^3 + 2^4 - 2^5 + \cdots - 2^{15}$

5. Simplify the following expression: $1 + x + x^2 + x^3 + \cdots + x^{100}$

Don't forget to take the ruler and compass with you for the next class.