

## Math 6: Homework 2.1

### Arithmetic Sequences

#### Arithmetic sequence

A sequence of numbers (typically but not always integers) is an arithmetic sequence if the difference between consecutive terms is the same number, the **common difference**, let's call it  $d$ .

For example, let's consider the sequence: 1, 5, 9, 13, 17, ...

The first term in the sequence is  $a_1 = 1$ , the second is  $a_2 = 5$ , and so on. The difference is  $d = 4$ .

What is the  $n^{\text{th}}$  term? For example what is  $a_{100}$ ?

$$a_1 = 1$$

$$a_2 = a_1 + d = 1 + 4 = 5$$

$$a_3 = a_2 + d = (a_1 + d) + d = a_1 + 2d = (1 + 4) + 4 = 1 + 2 \times 4 = 9$$

$$a_4 = a_3 + d = (a_2 + d) + d = ((a_1 + d) + d) + d = a_1 + 3d = 1 + 3 \times 4 = 13$$

....

$$a_n = a_1 + (n - 1)d$$

$$\text{So } a_{100} = a_1 + 99d = 1 + 99 \times 4 = 397$$

#### Property of an arithmetic sequence

A property of an arithmetic sequence is that any term is the arithmetic mean of its neighbors.

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

#### Sum of an arithmetic sequence

$$S = a_1 + a_2 + a_3 + \dots + a_n = n \cdot \frac{a_1 + a_n}{2}$$

**AMC 8 announcement:**

*The contest will be given on Th, Jan 18, 2024 (tentatively at 6pm).*

*If you want to register, please fill this form:*

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**Problems**

1. What are the first 2 terms of the arithmetic sequence  $a_1, a_2, -9, -2, 5, \dots$ ?
2. Find the common difference  $d$  in an arithmetic sequence if the 9-th term is 18 and the 11-th term is 44.
3. Find the sum of the first 100 terms if  $a_1 = -1$  and  $d = 1$ .
4. Find the sum of the first 1000 odd numbers.
5. Find the following sum:

$$2 + 4 + \dots + 2024$$

6. Prove that, given any arithmetic sequence, if I multiply each term by the same number and then add the same number to each term, the result is still an arithmetic sequence.
7. Simplify the following expression:

$$\frac{2}{\frac{1}{1 - \frac{1}{3}} - 1} \div \frac{\frac{1}{2}}{\frac{2}{3} - \frac{1}{4}}$$

8. In a given arithmetic progression, the first term is 6, and the 87-th term is 178. Find the common difference of this arithmetic progression and give the value of the first five terms.
9. The 3-rd term of the arithmetic progression is equal to 1. The 10-th term of it is three times as much as the 6-th term. Find the first term and the common difference. (**Hint:** Use the formula for the  $n$ -th term of the progression and write what is given in the problem using this formula.)
10. \*The sum of the first 20 terms of an arithmetic progression is 200, and the sum of the next 20 terms is -200. Find the sum of the first hundred terms of the progression.