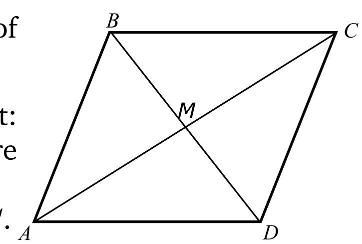
MATH 5: HOMEWORK 23
GEOMETRY 3.

1. Solve the equation $3x + 3 = \frac{1}{2}x + 13$.

2. (a) Explain why in a rectangle, opposite sides are equal.
(b) Show that a diagonal of a rectangle cuts it into two congruent triangles.

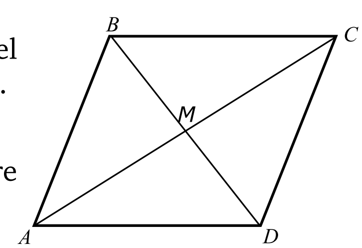
3. Let $ABCD$ be a parallelogram, and let M be the intersection point of the diagonals.

(a) Show that triangles $\triangle AMB$ and $\triangle CMD$ are congruent. [Hint: use the theorem proved in class, that the opposite sides are equal, and ASA.]
(b) Show that $AM = CM$, i.e., M is the midpoint of diagonal AC.



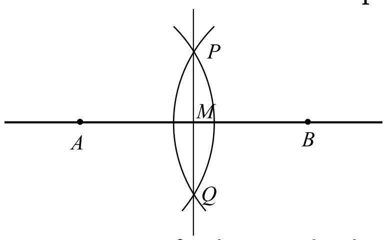
4. Let $ABCD$ be a quadrilateral such that sides AB and CD are parallel and equal (but we do not know if the sides AD and BC are parallel).

(a) Show that triangles $\triangle AMB$ and $\triangle CMD$ are congruent.
(b) Show that sides AD and BC are indeed parallel and therefore $ABCD$ is a parallelogram.



5. The following method explains how one can find the midpoint of a segment AB using a ruler and compass:

- Choose radius r (it should be large enough) and draw circles of radius r with centers at A and B.
- Denote the intersection points of these circles by P and Q. Draw the line PQ.
- Let M be the intersection point of lines PQ and AB. Then M is the midpoint of AB.

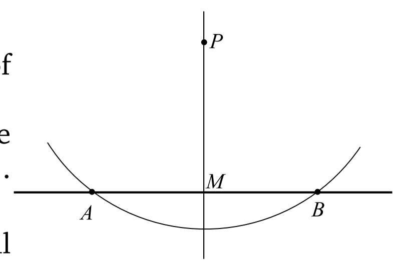


Can you justify this method, i.e., prove that so constructed point will indeed be the midpoint of AB? You can use the defining property of the circle: for a circle of radius r, the distance from any point on this circle to the center is exactly r. [Hint: $APBQ$ is a rhombus, so we can use problem 4 from the previous HW.]

6. The following method explains how one can construct a perpendicular from a point P to line l using a ruler and compass:

- Choose radius r (it should be large enough) and draw circle of radius r with center at P.
- Let A, B be the intersection points of this circle with l. Find the midpoint M of AB (using the method of the previous problem). Then MP is perpendicular to l.

Can you justify this method, i.e., explain why so constructed MP will indeed be perpendicular to l?



7. Let $ABCD$ be a parallelogram, and let BE, CF be perpendiculars from B, C to the line AD.

(a) Show that triangles $\triangle ABE$ and $\triangle DCF$ are congruent.

(b) Show that the area of parallelogram is equal to height$\times$base, i.e. $BE \times AD$.