

MATH 5: MATH BATTLE - PROBABILITY!

1. In a group of 100 students, 28 speak Spanish, 30 speak German, 42 speak French; 8 students speak Spanish and German, 10 speak Spanish and French, 5 speak German and French and 3 students speak all 3 languages. How many students do not speak any one of the three languages?
[Note: when it says that 28 students speak Spanish, this includes the 8 who speak Spanish and German; similarly for all other combinations.]

2. Suppose I have a standard die (6 faces on a cube, numbered 1 through 6).
 - (a) What is the probability that when we roll the die once, the number will be less than 5?
 - (b) What is the probability that when we roll the die once, the number will be less than 7?
 - (c) What is the probability that when we roll the die twice, at least one result will be a 6?
 - (d) What is the probability that when we roll the die twice, at least one result will be a 7?
 - (e) What is the probability that when we roll the die three times, all the results will be odd?

3. A cat is sitting on the table and a tortoise is crawling on the floor directly beneath it. The distance from the cat's ears to the top of the tortoise's shell is 170 cm. Ianni switched his pets around. Now the distance from the cat's ears to the top of the tortoise's shell is 130 cm. What is the height of the table?

4.
 - (a) What is the probability that if we roll 2 dice, the sum will be at most 7?
 - (b) A and B are playing the following game. They roll 2 dice; if the sum is at most 7, A wins, and B pays him \$1. Otherwise A loses and he pays to B \$1. Would you prefer to play for A or for B in this game?
 - (c) How to adjust the payments to make this game fair?

5. (a) What is the probability that if we roll 3 dice, all the numbers will be different?
 (b) A and B are playing the following game. They roll 3 dice; if all numbers are different, A wins, and B pays him \$2. Otherwise A loses and he pays to B \$3. Would you prefer to play for A or for B in this game?
 (c) How to adjust the payments to make this game fair?
6. (a) Two numbers are randomly chosen among 1, 2, and 3, one after the other (repeats are allowed). What is the chance that both numbers are the same?
 (b) Two numbers are randomly chosen among 1, 2, and 3, one after the other. What is the chance that they will be in strictly increasing order? (Strictly increasing means the second number must be greater than the first, they are not allowed to be equal.)
7. Suppose that you visit the Island of Knights (always tells the truth) and Knaves (always lies) because you have heard a rumor that there is gold buried there. You meet a native and you wish to find out from him whether there really is gold there, but you don't know whether he is a knight or a knave. You are allowed to ask him only one question answerable by 'yes' or 'no'. What would you ask?
8. Alex must walk through the 3x3 grid below left, moving either horizontally or vertically from cell to cell, and not entering any cell more than once. Below right shows one possible path. If Alex writes down the digits she passes in order during this path, he would get the number 84937561.

1	8	4
6	3	9
5	7	2

↑ 1	8	4
6	3	9
5	7	2