MATH 5
BEGINNING PROBABILITY – 2.

Question: we roll two dice. What is the probability of rolling a 5 and a 6?

Answer: There are two ways of getting a 5 and a 6: as pair $(5, 6)$ (5 on die number 1, 6 on die number 2) or as $(6, 5)$ (6 on die number 1, 5 on die number 2). Thus, the answer is $\frac{2}{36}$.

Question: we roll two dice. What is the probability of getting sum of two numbers equal to 4?

Answer: there 3 ways of getting sum 4: $(1, 3)$, $(2, 2)$, $(3, 1)$. Thus the probability is $\frac{3}{36} = \frac{1}{12}$.

PRODUCT RULE

Question. If toss a coin 10 times, what is the probability that all will be heads?

Answer. The probability of getting heads on one toss is 1/2. So, the probability of all heads in 10 tosses is $\left(\frac{1}{2}\right)^{10} = \frac{1}{2^{10}}$ (using calculator, one can compute that it is $1/1024 \approx 0.001$, or $1/10$ of 1%).

Question. If toss a coin 10 times, what is the probability that all will be tails?

Answer. The same as above.

Question. If we toss a coin 10 times, what is the probability that **at least one** will be heads?

Answer. Unfortunately, there are very many combinations which give at least one heads. In fact, it is easier to say which combinations **do not** give at least one heads: there is exactly one such combination, all tails; probability of getting this combination is, as we computed, $1/2^{10} = \frac{1}{1024}$. The remaining combinations will give at least one heads; thus probability of getting at least one heads is $1 - \frac{1}{1024} = \frac{1023}{1024} \approx 0.999$.

PERCENTAGES AND FRACTIONS

So far we have mostly expressed probabilities as fractions. They can also be written as decimal numbers (between 0 and 1): for example, $\frac{1}{5} = \frac{2}{10} = 0.2$. It is also common to express probabilities as percentages: by definition,

$$1\% = \frac{1}{100} = 0.01$$

so $x\% = \frac{x}{100}$. For example, $3\% = \frac{3}{100} = 0.03$, and $1.5\% = \frac{1.5}{100} = \frac{15}{1000} = 0.015$.

This conversion is necessary when you multiply probabilities as the following example shows:

Question. The probability of winning in a certain game is $p = 5\%$. What is the probability of winning two times in a row?

Answer. According to multiplication rule it is $p \times p = p^2$. However, the answer $5\% \times 5\% = 25\%$ is wrong. Correct answer is $\frac{5}{100} \times \frac{5}{100} = \frac{25}{10{,}000} = 0.0025$.

To convert from decimals to percent, multiply by 100:

$$p = (p \times 100)\%$$

For example, $\frac{1}{5} = 0.2 = (0.2 \times 100)\% = 20\%$

HOMEWORK

1. If we roll two dice, what is the probability that the product of two numbers is a multiple of 2?

2. Recall that a roulette has 37 slots: 0 through 36. Among slots 1–36, half are red, the other half black (zero has no color). What is the probability of
   (a) getting a red (on a single run of roulette)
   (b) getting a red, then black, then 0 (on 3 successive runs)
   (c) getting red 15 times in a row?
   (d) getting this sequence of colors: RRRBRBRBBRBBRBR (also of length 15)?

3. A hunter is shooting ducks. Probability of hitting a duck with one shot is $p = 1/3$.
   (a) What is the probability of missing the duck (with one shot)?
   (b) He makes 5 shots. What is the probability that he misses all five?
   (c) What is the probability that out of 5 shots, he will hit at least once? Will this probability double if he makes 10 shots? (You can use the calculator for computing the answers)
   (d) What is the probability that out of 5 shots, he will hit exactly once? Will this probability double is he makes 10 shots?

4. Supposing that there are equal chances of a boy or a girl being born, what is the probability that at least one of the first five babies born next Saturday morning at the St. Charles Hospital will be a girl? That all five will be girls?

5. At a fair, they offer you to play the following game: you are tossing small balls in a large crate full of empty bottles; if at least one of the balls lands inside a bottle, you win. Unfortunately, it is really impossible to aim, so the game is just a matter of luck (or probability theory): every ball you toss has a 20% probability of landing inside the bottle.
   (a) If you are given three balls, what is the probability that all three will be hits? That all three will be misses? That at least one will be a hit?
   (b) Same questions for five balls.

6. In a game, each student's name is written on identical slips of paper and put in a bag. Then a slip is randomly selected from the bag. Here is the composition of the class:

Grade	4^{th}	5^{th}	6^{th}
Number of students	9	5	11

If the teacher randomly selects a slip with a name, replaces it back to the bag, and then selects another name, what is the probability that both students chosen are in the 5^{th} grade?