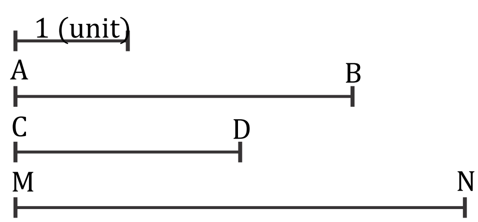
Math 5a. Classwork15.

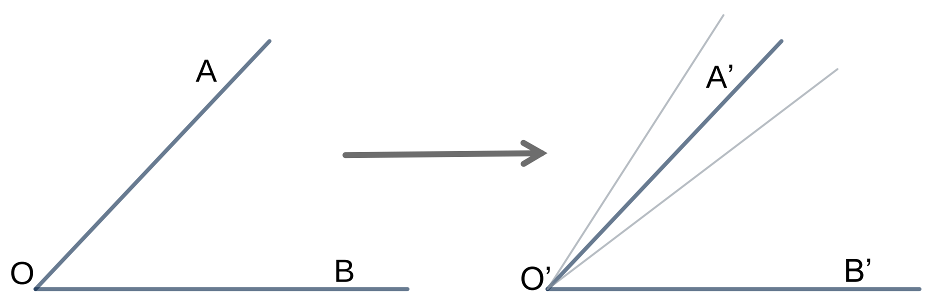
Congruent and non-congruent segments. Two segments are congruent if they can be laid on onto the other so that their endpoints coincide. Suppose that we put the segment [AB] onto the segment [CD] (pict. below) by placing the point A at the point C and aligning the ray [AB) with the ray [CD). If, as the result of this, the points B and D merge, then the segments [AB] and [CD] are congruent, or equal. Otherwise, they are not congruent, and the one which makes a part of the other is considered smaller.

We can introduce the concept of sum of several segments, we can subtract one segment from another. On the picture below, each segment contains several unit segments. Using compass find the length of each segment (in the unit segments).



How to construct the segment equal to another segment?

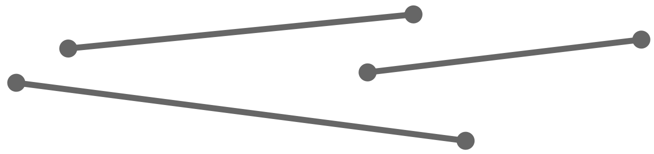
In the very similar way, we can define when two angles are congruent. Two angles are congruent if by moving one of them it is possible to superpose it with the other. The point O should be superposed with point O', ray OB is coincided with ray O'B'. If the ray OA will superpose with ray O'A' then 2 angles are congruent.



Sufficient evidence for congruence between two triangles can be shown through the following comparisons:

How many parameters are defining a triangle?

Can we construct a triangle by three sides?

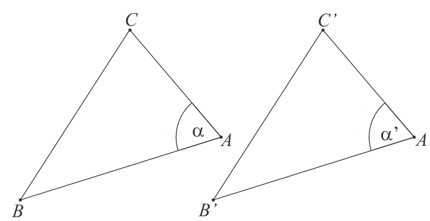


Can we construct a triangle with 2 known sides and an angle between them? Draw a triangle with sides 4 cm, 5 cm and an angle 30° between these two sides.

Side and two angles adjacent to the side?

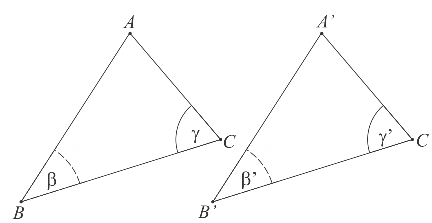
Three angles? Two sides and an angle, not adjacent to the side?

- **SAS** (Side-Angle-Side): If two pairs of sides of two triangles are equal in length, and the included angles are equal in measurement, then the triangles are congruent.
- **SSS** (Side-Side-Side): If three pairs of sides of two triangles are equal in length, then the triangles are congruent.
- **ASA** (Angle-Side-Angle): If two pairs of angles of two triangles are equal in measurement, and the included sides are equal in length, then the triangles are congruent.



SAS (Side-Angle-Side).
ABC and A'B'C' are two triangles such that

$AC = A'C', AB = A'B'$, and $\angle A = \angle A'$ We need to prove that these triangles are congruent. Superimpose ΔABC onto $\Delta A'B'C'$ in such a way that vertex A would coincide with A', the side AC would go along $A'C'$, and side AB would lie on the same side of $A'C'$ as $A'B'$. Since AC is congruent to $A'C'$, the point C will merge with point C'., due to the congruence of $\angle A$ and $\angle A'$, the side AB will go along $A'B'$ and due to the congruence of these sides, the point B will coincide with B'. Therefor the side BC will coincide with $B'C'$.



ASA (Angle-Side-Angle) ABC and $A'B'C'$ are two triangles, such that

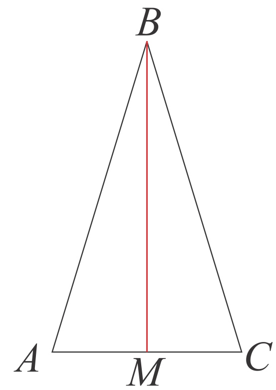
$\angle C = \angle C'$, $\angle B = \angle B'$, and $BC = B'B'$.

We need to prove, that these triangles are congruent. Superimpose the triangles in such a way that point C will coincide with point C', the side CB would go along

$C'B'$ and the vertex A would lie on the same side of $C'B'$ as A'. Then: since CB is congruent to $C'B'$, the point B will merge with B', and due to congruence of the angle $\angle B$ an $\angle B'$, and $\angle C$ and $\angle C'$, the side BA will go along B'A', and side CA will go along C'A'. Since two lines can intersect only at 1 point, the vertex A will have merge with A'. Thus, the triangles are identified and are congruent.

Theorem. In isosceles triangle the bisector passed to the base (in isosceles triangle the base is the side different from two equal sides) is a median and an altitude as well.

Let the triangle ΔABC be an isosceles triangle, such that $AB = BC$, and BM is a bisector. We need to prove that BM is a median and an altitude, which means that $AM = MC$ and angle $\angle BMC$ is a right angle.



BM is a bisector, so $\angle ABM$ and $\angle MBC$, the triangle ΔABM is an isosceles triangle, so $AB = BC$ and the segment MB is common side for triangles ΔABM and ΔMBC. Based on the Side-Angle-Side criteria, the triangles ΔABM and ΔMBC are congruent. Therefore, $AM = MC$ (BM is a median), angles $\angle A$ and $\angle C$ are congruent. (Isosceles triangle has equal angles adjacent to the base).

$\angle A + \angle B + \angle C = 180° = 2\angle A + \angle B \Rightarrow 90° = \angle A + \frac{1}{2}\angle B$ but for the triangle ABM (as well as for MBC), $\angle A + \frac{1}{2}\angle B + \angle BMA = 180°$, therefore $\angle BMA = 90°$and BM is also an altitude.

Exercise.

On one side of an angle $\angle A$, the segments AB and AC are marked, and on the other side the segments $AB' = AB$ and $AC' = AC$. Prove that the lines BC' and $B'C$ met on the bisector of the angle $\angle A$.