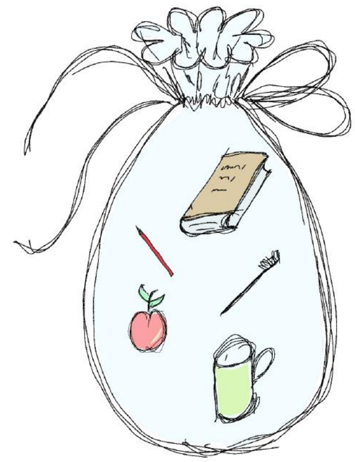
Classwork 11.

Sets and numbers.



I put a pencil, a book, a toothbrush, a coffee mug, and an apple into a bag. Do all these objects have something in common?

A set is a collection of objects that have something in common.

Can we call this collection of items a set? What is a common feature of all these objects? They are all in the bag, where I put them.

There are two ways of describing, or specifying, the members of a set. One way is by listing each member of the set, as we did with our set of things.

I can create, for example, a set of my favorite girls' names (F):

$\boldsymbol{F} = \{Mary,\ Kathrine, Sophia\}$.

The name of the set is usually indicated by a capital letter, in my case is F, list of members of the set is included in curved brackets. I can also create a set of all girls' names (N):

$\boldsymbol{N} = \{n |\ n = girl's\ name\}$

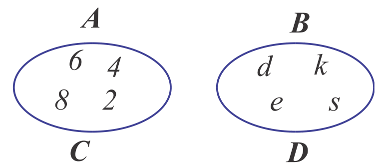
Of cause, I can't itemize all possible names, there are too many of them, we don't have enough space here for that, but I can describe the common feature of all members of the set – they are all girl's names. In the mathematical phrase above (1) I described the set, I call it N, which contains an unknown number of items, (we really don't know how many names exist), I use variable n to represent these items, girls' names. I show it by the phrase $n = girl's\ name$.

In our everyday life we use the concept of set quite often, we even have a special word for some sets, for example, the word "family" indicates a set of people, connected to each other, "class of 2020" – means all students who will graduate in 2020 and so on. Can you give more examples of such words and expressions?

When a set is created, about any object can be said that this object belongs to the set or not. For example, name "Emily" does not belong to the set F, number 2 also doesn't belong to this set. But "Emily" belongs to the set N, because it is a girl's name, number 2 is not a name.

Let's consider two examples of sets:

Sets A and B are created by listing their items explicitly:



Venn diagrams

A = {2, 4, 6, 8}, B = {d, e, s, k}.

Sets C and D are created by describing the rules according with which they were created:

C is the set of four first even natural numbers.

D is the set of letters of the word "desk".

If we look closer on our sets A and C, we can see that all elements of set A are the same as elements of set C (same goes for sets B and D).

$$\boldsymbol{A} = \boldsymbol{C} \text{ and } \boldsymbol{B} = \boldsymbol{D}$$

Two sets are equal if they contain exactly the same elements.

If a set A contains element '2', then we can tell that element '2' belongs to the set A. We have a special symbol to write it down in a shorter way: $2 \in A$, $105 \notin A$. (What does this statement mean?)

When a set is created, we can say about any possible element does it belong to the given set or not.

Let's define several sets:

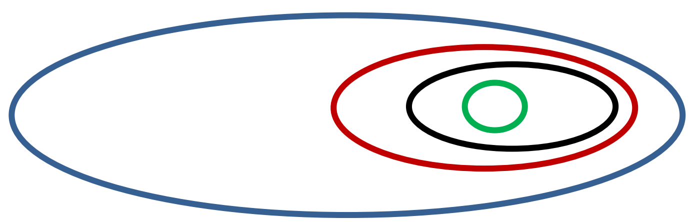
Set W will be the set of all words of the English language.

Set N will be the set of all nouns existing in the English language.

Set Z will be the set of all English nouns which have only 5 letters.

Set T={"table"}.

On a Venn diagram name all these sets:



If all elements of one set at the same time belong to another set then we can say that the first set is a subset of the second one. We have another special symbol to write this statement in a shorter way: $\subset$.

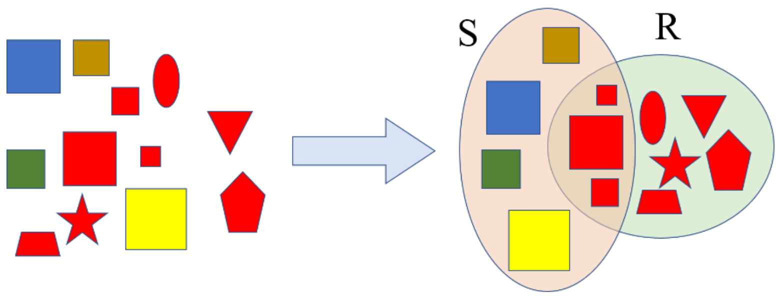
$T \subset Z \subset Y \subset W$

Problem:

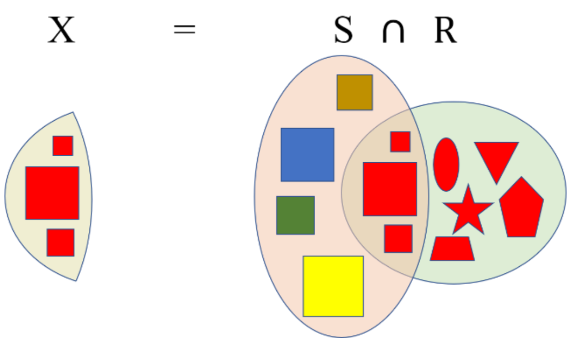
Draw Venn diagrams for the following sets:

Set A is the set of all cities of the United States. Set B is the set of all cities of the New-York state. Set C contains only one element, C = {Stony Brook}. Set D = {Paris(France), London(GB), Deli(India)}. Write the relationship between these sets.

When several sets are defined it can happen, that in accordance with all the rules we have implied, several objects can belong to several sets at the same time. For example, on a picture set S is a set of squares and a set R is a set of red shapes.



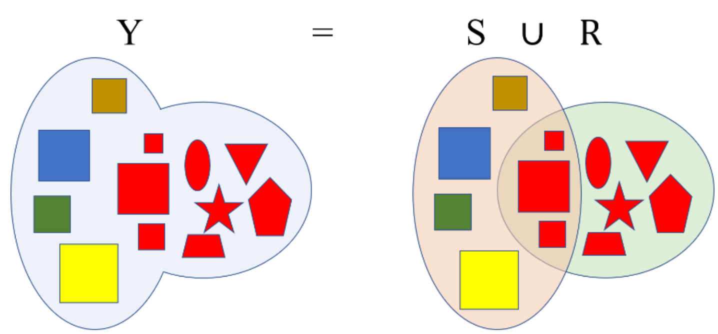
A few figures are squares and they are red, therefore they belong to both sets. Thus, we can describe a new set X containing elements that belong to the set S as well as to the set R. The new set was constructed by determining which members of two sets have the features of both sets. This statement also can be written down in a shorter version by using a special symbol ∩. Such set X is called an intersection of sets S and R.



$\boldsymbol{X = S \cap R}$

Another new set can be created by combining all elements of either sets (in our case S and R). Using symbol ∪ we can easily write the sentence: Set Y contains all elements of set S and set R:

$\boldsymbol{Y = S \cup R}$

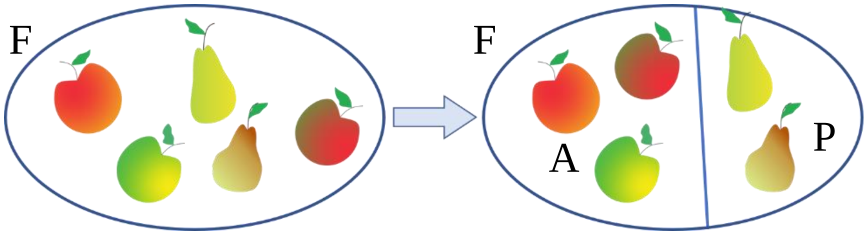


Such set Y is called a union of set S and R.

Set which does not have any element called an empty set (in math people use symbol ∅). For example, the set of polar bears living in the Antarctica is an empty set (there is no polar bears in Antarctica).

$\in$	element belongs to a set	$\not\subset$	one set is not a subset of another set
$\notin$	element does not belong to a set	$\cap$	intersection of two sets
$\subset$	one set is a subset of another set	$\cup$	union of two sets
$\emptyset$	empty set		

There is a basket with fruits on the table. It contains 3 apples and 2 pears. The set of fruits in the basket can be divided into two sets so, that these two sets do not intersect (apple can't be a pear at the same time), also in there is no any other elements in the set:



In this case we can say, we "classified" the set, we did the classification of the elements of the set.

$$\mathrm{A} \subset \mathrm{F}, \qquad \mathrm{P} \subset \mathrm{F}, \qquad \mathrm{A} \cap \mathrm{P} = \emptyset, \qquad \mathrm{A} \cup \mathrm{P} = \mathrm{F}$$

WE can now introduce the operation of addition and subtraction of sets.

$$\mathrm{F} = \mathrm{A} + \mathrm{P}, \qquad \mathrm{A} = \mathrm{F} - \mathrm{P}, \qquad \mathrm{P} = \mathrm{F} - \mathrm{A}$$

Exercise:

1. Give examples of several members of the following sets:

   Example:

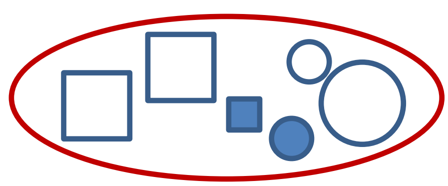
   $M = \{x |\, x = mammals\}$

   x can be a lion, a whale, a bat…

   a. $K = \{y |\, y = letter\ of\ the\ english\ alfabet\}$
   b. $M = \{x |\, x = flower\}$
   c. $X = \{m |\, m = even\ number\}$
   d. $P = \{k |\, k = color\}$

2. $A = \{2, 5, 0, 1\}$, $B = \{2, 0, 1\}$, $C = \{0, 2, 5, 1\}$, and $D = \{2, 0, 5, 4, 1\}$
   Which sets are equal?

3. How this set can be classified? Name the sets with capital letters, write all corresponding statements.



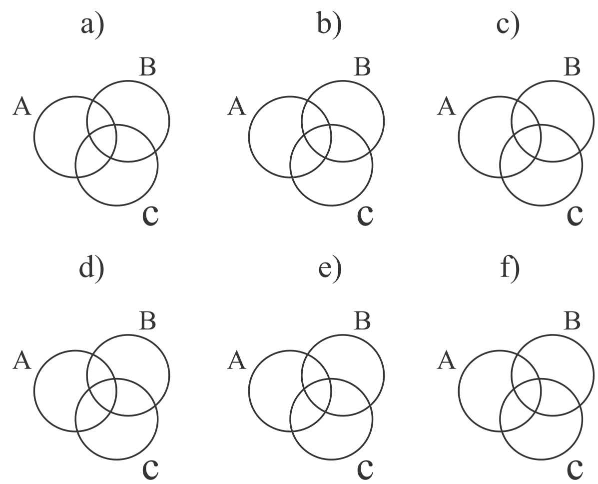
4. There are 21 students in a Math class. 10 students like apples and 15 students like pears. Show that there are some students, who like both apples and pears. Is it possible to determine if there any students who do not like apples and do not like pears? Explain your answer.

Assume, that each student likes at least one of the fruits. (This means that each student likes either apples, or pears, or both). How many students do like both pears and apples?

5. The same Math class (with 21 students) forms a soccer team and a basketball team. Every student signs up for at least one team: 12 students play only soccer; 2 students play both soccer and basketball; How many students play basketball only?

6. Students who participated in math coopetition had to solve 2 problems, one in algebra and another in geometry. Among 100 students 65 solved algebra problem, 45 solved geometry problem, 20 students solved both problems. How many students didn't solve any problem at all?

7. On the diagrams of sets A, B, and C put 2 elements so that (just draw 2 points, or put any two letters).
    a. each set contains 2 elements
    b. set A contains 2 elements, set B contains also 2 elements, and set C contains 1 element.
    c. set A contains 2 elements, sets B and C contain 1 element each
    d. set A contains 2 elements, set B contains 1 element, and set C is an empty set
    e. set A contains 2 elements, set B contains 2 elements, and set C is an empty set
    f. each set contains 1 element



Geometry.

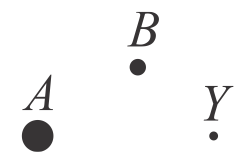
A **definition** is a statement of the meaning of a something (term, word, another statement).

Desk *noun*
noun: **desk**; plural noun: **desks**

1. a piece of furniture with a flat or sloped surface and typically with drawers, at which one can read, write, or do other work.
    - Music
      a position in an orchestra at which two players share a music stand.
      "an extra desk of first and second violins"
    - a counter in a hotel, bank, or airport at which a customer may check in or obtain information.
      "the reception desk"

In mathematics everything (mmm,,,, almost everything) should be very well defined. In our real life, it is also very useful and convenient to agree about terms and concepts, to give them a definition, before starting using them just to be sure that everybody knows what they are talking about.   Now we move to geometry.

Can we give a definition to a point? Can we clearly define what a point is? What a line is? What a plane is? Mathematicians decided do not define terms "point", "straight line", and "plane" and to rely upon intuitive understanding of these terms.

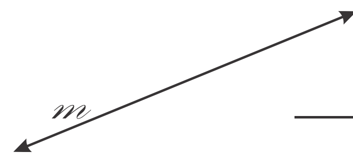


Point (an undefined term).

In geometry, a point has no dimension (actual size), point is an exact location in space. Although we represent a point with a dot, the point has no length, width, or thickness. Our dot can be very tiny or very large and it still represents a point. A point is usually named with a capital letter.

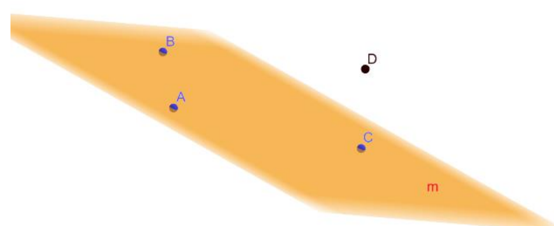
Line (an undefined term).

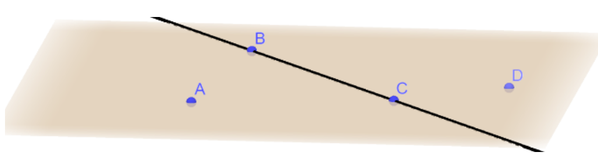
In geometry, a line has no thickness but its length extends in one dimension and goes on forever in both directions. Unless otherwise stated a line is drawn as a straight line with two arrowheads indicating that the line extends without end in both directions (or without them). A line is named by a single lowercase letter, m for example, or by any two points on the line, $\overleftrightarrow{AB}$ or AB.



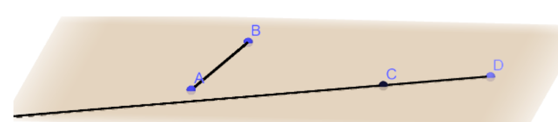
Plane (an undefined term).

In geometry, a plane has no thickness but extends indefinitely in all directions. Planes are usually represented by a shape that looks like a parallelogram. Even though the diagram of a plane has edges, you must remember that the plane has no boundaries. A plane is named by a single letter (plane p) or by three non-collinear points (plane ABC).





Points can belong to the plane or can be outside of the plane. On a plane, points can belong to the straight line, or can be positioned on ether half-plane.



A set of all points of a straight line between two specific points. These points are called endpoints.

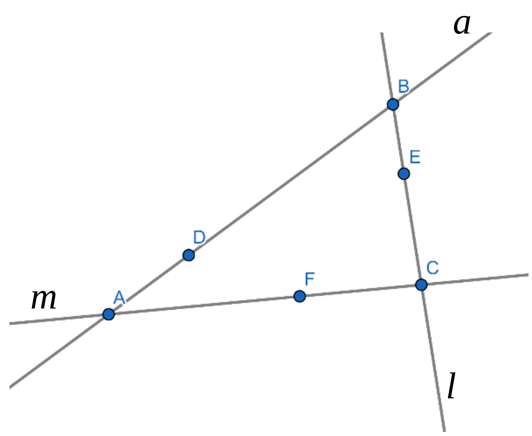
A ray is a part of a straight line consisting of a point (endpoint) and all points of a straight line at one side of an endpoint. Ray is named by   endpoint and any other point, ray $\overrightarrow{AB}$ or AB (where A is an endpoint)

Exercises:

1. Draw a segment 2 cm long, 5 cm long, a square with the side 4 cm. (use ruler, pencil).
2. Draw two line segments AB and CD in such way that their intersect
   a. by a point
   b. by a segment
   c. don’t intersect at all.

3. Using a ruler draw a straight line, put on it 3 points, *A*, *B*, and *C* so that 2 rays are formed, *BC* and *BA*.

4. Draw two rays AB and CD in such way that their intersect
   d. by a point
   e. by a segment
   f. by a ray
   g. don’t intersect at all.



5. Through which points does the lime *m* pass?
   Through which points does the lime *a* pass?
   What is the intersection of the lines *m* and *l*?
6. Mark 2 points. How many different lines can be drawn through these two points?
7. Mark three points. How many lines can be drawn through three points?
   Consider all possible solution.
8. Mark four points. How many lines can be drawn through four points?