Math 5a. Classwork 5.

Exercises:

1. Add the same digit to the number 10 on the right and left so that the resulting four-digit number is divisible by 12.

2. Write in a row the first 10 prime numbers. How do you cross out 6 digits to get the largest possible number?

   First 10 prime numbers:
   2357111317192329 It's a 16-digit number. If 6 digits are removed, it will become a 10-digit number.
   2357111317192329 -> 7317192329

3. In which number system is the equation 3 · 4 = 10 true?

   First 2-digit number in that system should be the product of 3 and 4, so it's 12 in decimal. In based 12 system.

4. Can you find out which numbers are multiplied?

$$\begin{array}{r} * * \\ \times\ \underline{* *} \\ * * \\ +\ \underline{* * *\ } \\ 9 * * * \end{array}$$

   We can say that both numbers are greater than or equal to 90 (one can be 90, but the other one should be greater than 90). The second number can't have a last digit other than one; if the last digit is greater than 1, the first line will have three digits. The last digit of the first number can't be anything but 9. If it's not 9, the final product will start with 8, not 9.

   Answer: 99·91=9009

5. Can you find out which numbers are multiplied?

$$\begin{array}{r} * * \\ \times\ \underline{* *} \\ * * \\ +\ \underline{* 7\ } \\ * * * * \end{array}$$

   (Hint: The answer should start with 10 and second line of the sum should be 97 (think why?).

6. Write number 295 in 8-based (octal) system.
7. Write number 432 in 8-based (octal) system.

   $432 = 384 + 48 = 64 \cdot 6 + 8 \cdot 6 = 8^2 \cdot 6 + 8^1 \cdot 6 + 8^0 \cdot 0 = 660_8$

8. Write number 376_8 in decimal system.
9. Write number 735_8 in decimal system.

   $735_8 = 8^2 \cdot 7 + 8^1 \cdot 3 + 8^0 \cdot 5 = 64 \cdot 7 + 8 \cdot 3 + 1 \cdot 5 = 448 + 24 + 5 = 477$

10. Write number 10101010_2 in decimal system.

    $$\begin{aligned} 10101010_2 &= 2^7 \cdot 1 + 2^6 \cdot 0 + 2^5 \cdot 1 + 2^4 \cdot 0 + 2^3 \cdot 1 + 2^2 \cdot 0 + 2^1 \cdot 1 + 2^0 \cdot 0 \\ &= 128 + 32 + 8 + 2 = 170 \end{aligned}$$

11. Write number 11101111_2 in decimal system.

12. Write number 295 in binary system.

    $$\begin{aligned} 295 = 256 + 32 + 4 + 2 + 1 &= 2^8 + 2^5 + 2^2 + 2^1 + 2^0 \\ &= 2^8 \cdot 1 + 2^7 \cdot 0 + 2^6 \cdot 0 + 2^5 \cdot 1 + 2^4 \cdot 0 + 2^3 \cdot 0 + 2^2 \cdot 1 + 2^1 \cdot 1 + 2^0 \cdot 1 \\ &= 100100111_2 \end{aligned}$$

13. Write number 111 in binary system.
14. Write number 111_2 in decimal system.

15. Two ladybugs crawled down the wall from the ceiling to the floor. Upon reaching the floor, they crawled back up. The first fly crawled to both ends at the same speed, while the second, though it climbed twice as slowly as the first, descended twice as fast. Which of the ladybug will reach back first? Which fly has a higher average speed?

    This problem can be solved formally and informally. Informally:

    Because, the time needed for the second ladybug to go up from the floor to the ceiling is the time the first ladybug will go to both ways. So, first will go first.

    Formal way:

    Let h be the height of the wall, v be the speed of the first ladybug. Then the time of the journey of the fist ladybug will be

    $$t_1 = \frac{2h}{v}$$

    Time needed for the second ladybug is

$$t_2 = \frac{h}{2v} + \frac{h}{\frac{1}{2}v} = \frac{h}{2v} + \frac{2h}{v} = \frac{h}{2v} + \frac{4h}{2v} = \frac{h + 4h}{2v} = \frac{5h}{2v} = \frac{5}{2}\frac{h}{v}$$

$$\frac{t_1}{t_2} = \frac{2h}{v} : \frac{5h}{2v} = \frac{2h}{v} \cdot \frac{2v}{5h} = \frac{4}{5}$$

This not only shows that the first ladybug is faster but also quantifies how much faster it is.

16. Two people simultaneously set out from A to B. The first one rode a bicycle, while the second one traveled by car at a speed five times greater than the first. Halfway to the destination, the car experienced an accident, and the motorist continued the remaining journey on foot at a speed half that of the bicyclist. Who arrived in B first?
    This problem is similar to the previous one. Informally, it will take the driver the same time to walk from the place of accident to B, as for cyclist to go from A to B, but the cyclist started from A before the accident occurred. So, cyclist will reach B firs.
    Formal way: Let S be a distance between A and B, v be the speed of the cyclist. The cyclist's time to reach B is:

$$t_1 = \frac{S}{v}$$

The driver's time is:

$$t_2 = \frac{\frac{1}{2}S}{5v} + \frac{\frac{1}{2}S}{\frac{1}{2}v} = \frac{\frac{1}{2}S}{5v} + \frac{S}{v} = \frac{S}{10v} + \frac{S}{v} = \frac{S}{10v} + \frac{10S}{10v} = \frac{11S}{10v}$$

$$t_1 : t_2 = \frac{S}{v} : \frac{11S}{10v} = \frac{S}{v} \cdot \frac{10v}{11S} = \frac{10}{11}$$

17. Two friends set out simultaneously from two villages towards each other, with the distance between the villages being 18 km. The first friend walked at a speed of 5 km/h, while the second one walked at 4 km/h. The first friend brought a dog with him, which ran at a speed of 8 km/h. The dog immediately ran towards the second man, met him, barked, turned around, and ran towards its owner at the same speed, and so on. This continued until the friend met. How many kilometers did the dog run?
    Dog will run back and force all the time till friends meet.

$$v = 5 + 4 = 9\ km/h$$

$$t = 18km : \frac{9km}{h} = 2\ hours$$

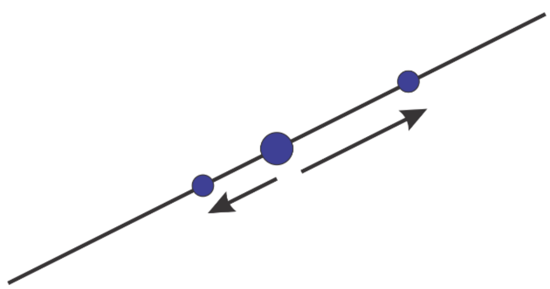
Speed of the dog is $8\ km/h$*, so dog was running for 2 hours at a speed of* $8\ km/h$*:*

$$S = 2h \cdot 8\frac{km}{h} = 16km$$

18. At 9 am, a motorboat set off upstream on the river, and at the moment of its departure, a ball was thrown from the boat into the river. At 9:15, the boat turned and started sailing downstream. At what time will the boat catch up with the ball, given that its own speed remained constant?

Informal way:

The speed of the river flow is the same for the boat and the ball. So, the distance between the ball and the boat increases for 15 minutes and then decreases at the same speed. Boat will catch up with the ball in 30 minutes after 9:00 am.



Formal way:

v_r *is the river flow (km/h)*

v_b *is the speed of the boat (km/h)*

In 15 minutes, the boat will be $0.25 \cdot (v_b - v_r)$ *kilometers away from starting point upstream, while ball will be* $0.25 \cdot v_r$ *kilometer downstream from the starting point. The total distance between them is*

$0.25 \cdot (v_b - v_r) + 0.25 \cdot v_r = 0.25 \cdot v_b - 0.25 \cdot v_r + 0.25 \cdot v_r = 0.25 \cdot v_b$ *kilometers.*

Speed of the flow is the same for both, the distance will reduce at the speed of the boat alone.

$$t = \frac{0.25 \cdot v_b}{v_b} = 0.25\ hour.$$

19. The bathtub fills with cold water in 6 minutes and 40 seconds and with hot water in 8 minutes. Additionally, if the plug is removed from a full bathtub, it drains in 13 minutes and 20 seconds. How much time will it take to completely fill the tub when both taps are open, but the plug is not inserted?

In one second $\frac{1}{6\cdot 60+40}=\frac{1}{400}$ *part of the bathtub will be filled with cold water, while the hot water tap will fill* $\frac{1}{8\cdot 60}=\frac{1}{480}$ *part of the bathtub and* $\frac{1}{60\cdot 13+20}=\frac{1}{800}$ *part will be drained.*

$$\frac{1}{400}+\frac{1}{480}-\frac{1}{800}=\frac{2}{800}-\frac{1}{800}+\frac{1}{480}=\frac{1}{800}+\frac{1}{480}=\frac{3}{2400}+\frac{5}{2400}=\frac{8}{2400}=\frac{1}{300}$$

$$t\cdot\frac{1}{300}=1;\quad t=1:\frac{1}{300}=300\ s.$$

Answer: 300 seconds or 5 minutes.

20. To bake 100 pancakes, Mom needs 30 minutes, while Dad needs 40 minutes. Son can eat 100 pancakes in an hour. Mom and Dad continuously make pancakes without stopping, while Son continuously eats them. After how much time from the beginning of this process will there be exactly 100 pancakes on the table?

21. 7 wolves eat 7 sheep in 7 days. How many days will it take for 9 wolves to eat 9 sheep?