Math 5a. Classwork 3.

If a number a in a power n is divided by the same number in a power m,

$$\frac{a^n}{a^m} = \frac{\underbrace{a \cdot a \cdot \ldots \cdot a}_{n\ times}}{\underbrace{a \cdot a \cdot \ldots \cdot a}_{m\ times}} = \left(\underbrace{a \cdot a \cdot \ldots \cdot a}_{n\ times}\right) : \left(\underbrace{a \cdot a \cdot \ldots \cdot a}_{m\ times}\right) = a^n : a^m = a^{n-m}$$

So, we can say that

$$a^{-1} = \frac{1}{a^1} = \frac{1}{a}; \qquad a^{-n} = \frac{1}{a^n};$$

Let's see how our decimal system of writing numbers works when we use the concept of exponent: $3456 \ = \ 1000 \cdot 3 + 100 \cdot 4 + 10 \cdot 5 + 1 \cdot 6 = 103 \cdot 3 + 102 \cdot 4 + 101 \cdot 5 + 100 \cdot 6$

The value of a place of a digit is defined by a power of 10 multiplied by the digit. Very large numbers can be written using this system, as well as very small numbers.

$$0.3 = \frac{1}{10} \cdot 3 = 10^{-1} \cdot 3;$$

$$0.456 = \frac{1}{10} \cdot 4 + \frac{1}{100} \cdot 5 + \frac{1}{1000} \cdot 6 = 10^{-1} \cdot 4 + 10^{-2} \cdot 5 + 10^{-3} \cdot 6$$

Scientists work with very large and very small things, from galaxies to viruses. They need to be able to write numbers, describing the object of interest, for example the distance between two galaxies or the diameter of a virus.

One of the most important numbers in the universe is the speed of light.

299 792 458 m / s. It's very convenient to represent it as a decimal starting with units and multiplied by a power of 10.

$$299\ 792\ 458\ m\ per\ s = 2.99792458 \cdot 10^8\ m\ p\ s.$$

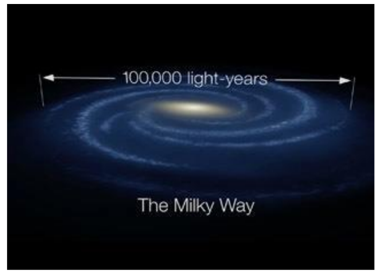
Let's convert the value to kilometers per hour. Each kilometer is 1000 meters, so we need to divide it by 1000:

$$3 \cdot \frac{10^8}{10^3} mps = 3 \cdot 10^{8-5} = 3 \cdot 10^5 km\ per\ s$$

In each hour there are 3600 seconds, or $3.6 \cdot 10^3$ seconds. To find out the speed of light in km per hour we now need to multiply the speed in seconds by $3.6 \cdot 10^3$

$$3 \cdot \frac{10^8}{10^3} mps = 3 \cdot 10^{8-5} = 3 \cdot 10^5 km\ per\ s = 3 \cdot 10^5 \cdot 3.6 \cdot 10^3 = 10.8 \cdot 10^8\ km\ p$$

The Milky Way galaxy has a diameter of 105,700 light years, so the light will travel from one end to the other through its center in 105700 years.



How far is one side from the other in the Milky Way in kilometers? 10.8·108$km\ p\ h$ ·105700 $years$.

How many hours in a year? $24 \cdot 365.25 = 8766 \approx 8.8 \cdot 10^3\ hours$

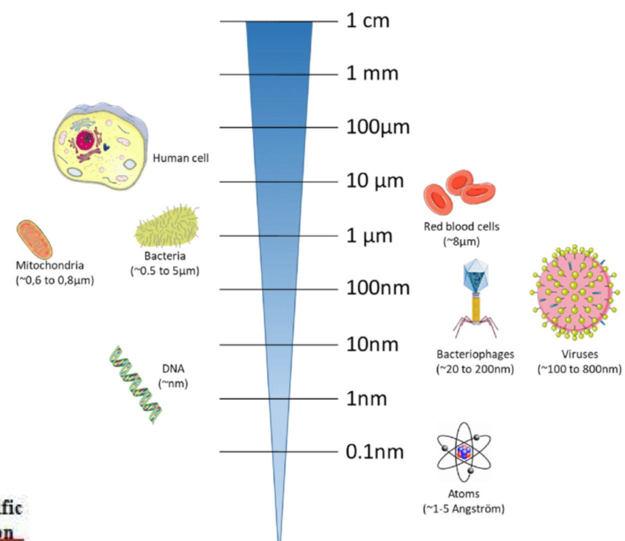
$$10.8 \cdot 10^8\ km\ p\ h\ \cdot 105700\ years \approx 1.08 \cdot 10^9\ km\ p\ h \cdot 8.8 \cdot 10^3 \cdot 1.06 \cdot 10^5 \approx 10 \cdot 10^{17} km$$
$$\approx 10^{18} km.$$

This way to write numbers is called scientific notation, it's used a lot in science for describing various objects, big and small. Let's take a look on the small things, like bacteria and viruses.

$$1\ cm = 0.01\ m = \frac{1}{100} m = \frac{1}{10^2} m = 10^{-2} m$$

$$1\ mm = 0.001\ m = \frac{1}{1000} m = \frac{1}{10^3} m = 10^{-3} m$$

$$1\ \mu m = 10^{-6} m; \qquad 1nm = 10^{-9} m$$



Bactria are between $0.5 - 1.5\ \mu m$

$0.5 \mu m = 0.5 \cdot 10^{-6} m = 5 \cdot 10^7 m$

Prefix	Symbol for Prefix		Scientific Notation
exa	E	1 000 000 000 000 000 000	10^{18}
peta	P	1 000 000 000 000 000	10^{15}
tera	T	1 000 000 000 000	10^{12}
giga	G	1 000 000 000	10^{9}
mega	M	1 000 000	10^{6}
kilo	k	1 000	10^{3}
hecto	h	100	10^{2}
deka	da	10	10^{1}
----	--	1	10^{0}
deci	d	0.1	10^{-1}
centi	c	0.01	10^{-2}
milli	m	0.001	10^{-3}
micro	µ	0.000 001	10^{-6}
nano	n	0.000 000 001	10^{-9}
pico	p	0.000 000 000 001	10^{-12}
femto	f	0.000 000 000 000 001	10^{-15}
atto	a	0.000 000 000 000 000 001	10^{-18}

Review.

1. In 2 hours, the car traveled 96 km, and the cyclist covered 72 km in 6 hours. By how many times was the car faster than the cyclist?

   The speed (how far the object can go in one unit of time), time of the travel, and the total distance are connected by the expression:

$$S(distance) = v\ (speed) \cdot t(time)$$

   To find out the speed of the car (how far it travels in 1 hour) :

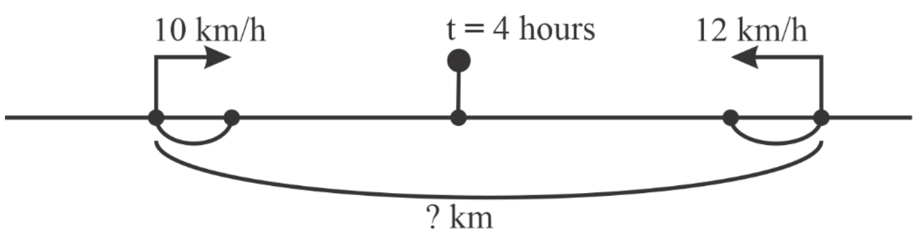
$$96\ km: 2h = 48\ km\ per\ h\ \left(\frac{km}{h}\right)$$

   To find out the speed of the cyclist:

$$72\ km: 6\ h = 12\frac{km}{h}$$

   $48: 12 = 4$ times the car is taster than the cyclist.

2. Two cyclists set off from two different towns towards each other at the same time. The speed of the first cyclist is 10 km/h, and the speed of the second cyclist is 12 km/h. They met after 4 hours. Determine the distance between the two towns.



   How fast the distance between these cyclists will be decreasing?

   In 1 hour, they will be closer by $10 + 12 = 22\ km$. The distance will be decreasing with the speed of both cyclist combined. They were traveling for 4 hours, so the distanceis

   $22\frac{km}{h} \cdot 4\ h = 88km$

3. The distance between Athos and Aramis, who are traveling on the same road, is 20 leagues. In an hour, Athos covers 4 leagues, and Aramis covers 5 leagues. What will be the distance between them after an hour?
4. I walk from home to school in 30 minutes, and my brother takes 40 minutes. How many minutes will it take for me to catch up to my brother if he left home 5 minutes before me?

5. Let's take a look on a sequence of numbers:

   $2, \quad 2^2, \quad 2^3, \quad 2^4, \ldots \quad 2^{12}$

   What are the last digits of this numbers? Can you tell the last digit of $2^{13}, \quad 2^{14}, \quad 2^{15}$?

   What about $2^{32}, \quad 2^{49}, \quad 2^{62}$?

   Can you tell what would be the last digit of 2022^{23}? 2025^{23}? 2023^{23}? 2026^{23}

6. Compare the following exponents:

   $a.\ 2^{10}\ \ and\ \ 10^3;$ $\qquad b.\ 10^{100}\ \ and\ 100^{10}$

   $c.\ 2^{300}\ \ and\ \ 200;$ $\qquad d.\ 31^{16}\ \ and\ 17^{20};$ $\qquad e.\ 4^{53}\ \ and\ 15^{45}$

7. Prove that

   $8^5 + 2^{11}$ is divisible by 17

   $9^7 - 3^{10}$ is divisible by 20

8. $x^5 < y^8 < y^3 < x^6$

   Where 0 should be placed?