Math 5a. Classwork 2.

1. Fill in the empty cell in the table:

dividend	a	29		46	94
divisor	b	7	9		9
quotient	c	4	7	3	
remainder	r		5	1	4

$$a = b \cdot c + r$$

2. One bottle can hold 5 liters of apple juice. How many 5-liter bottles are needed to pour 47 liters of juice into them?

3. The sum of two natural number is 45. First number gives a remainder of 4 when divided by 12, the second gives a remainder 5 when divided by 12. What are these numbers?

4. The sum of two natural number is 54. First number gives a remainder of 11 when divided by 17, the second gives a remainder 9 when divided by 17. What are these numbers?

5. The sum of two natural number is 48. First number gives a remainder of 14 when divided by 19, the second gives a remainder 15 when divided by 19. What are these numbers?

6. The sum of two numbers is 242, and when the larger of these numbers is divided by the smaller one, the quotient is 4, and the remainder is 22. Find the smaller of these numbers.

7. The number a is even. Can the remainder of the division of the number a by 6 be equal to 1? 3?

8. When dividing a natural number, a by 2, the remainder is 1, and when dividing it by 3, the remainder is 2. What will be the remainder when a is divided by 6?

Exponent.

Exponentiation is a mathematical operation, written as a^n, involving two numbers, the base a and the exponent n. When n is a positive integer, exponentiation corresponds to repeated multiplication of the base. In other words, a^n is the product of multiplying n bases:

$$a^n = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{n\ times}$$

In that case, a^n is called the n-th power of a, or a raised to the power n.
The exponent indicates how many copies of the base are multiplied together.

Properties of exponent.
Based on the definition of the exponent, a few properties can be derived.

$$a^n \cdot a^m = \underbrace{a \cdot a \dots \cdot a}_{n\ times} \cdot \underbrace{a \cdot a \dots \cdot a}_{m\ times} = \underbrace{a \cdot a \cdot a \dots \cdot a}_{n+m\ times} = a^{n+m}$$

$$(a^n)^m = \underbrace{a^n \cdot a^n \cdot \dots \cdot a^n}_{m\ times} = \underbrace{\underbrace{a \cdot a \cdot \dots \cdot a}_{n\ times} \cdot \dots \cdot \underbrace{a \cdot a \cdot \dots \cdot a}_{n\ times}}_{m\ times} = a^{n \cdot m}$$

If a number a in a power on n multiplied by the number a one more time, the total number of multiplied bases increased by 1:

$$a^n \cdot a = \underbrace{a \cdot a \cdot a \dots \cdot a}_{n\ times} \cdot a = \underbrace{a \cdot a \cdot a \cdot a \dots \cdot a}_{n+1\ times} = a^{n+1} = a^n \cdot a^1$$

In order to have the set of power properties consistent, any number in the first power is the number itself. In other words, $a^1 = a$ for any number a.
Also, a^n can be multiplied by 1:
$a^n = a^n \cdot 1 = a^{n+0} = a^n \cdot a^0$

In order to have the set of properties of exponent consistent, $a^0 = 1$ for any number a, but 0.
If there are two numbers a and b:

$$(a \cdot b)^n = \underbrace{(a \cdot b) \cdot \dots \cdot (a \cdot b)}_{n\ times} = \underbrace{a \cdot \dots \cdot a}_{n\ times} \cdot \underbrace{b \cdot \dots \cdot b}_{n\ times}$$
$$= a^n \cdot b^n$$

All these properties can be summarized:

1. $a^n = \underbrace{a \cdot a \cdot a \dots \cdot a}_{n\ times}$
2. $a^n \cdot a^m = a^{n+m}$
3. $(a^n)^m = a^{n \cdot m}$
4. $a^1 = a$, for any a
5. $a^0 = 1$, for any $a \neq 0$
6. $(a \cdot b)^n = a^n \cdot b^n$

Positive and negative numbers:

- A positive number raised into any power will result a positive number.
- A negative number, raised in a power, represented by an even number is positive, represented by an odd number is negative.

Exercises:

1. Continue the sequence:

$a.$ $1, 4, 9, 16 \dots$ $b.$ $1, 8, 27, \dots$ $c.$ $1, 4, 8, 16 \dots$

2. Write the following products as exponents:

Example:

$-2 \cdot 2 \cdot 2 \cdot 2 = -2^4;$ $(-2) \cdot (-2) \cdot (-2) \cdot (-2) = (-2)^4$

$a.$ $(-3) \cdot (-3) \cdot (-3) \cdot (-3);$

$b.$ $(-5m)(-5m) \cdot 2n \cdot 2n \cdot 2n;$

$c.$ $-3 \cdot 3 \cdot 3 \cdot 3;$

$d.$ $-5m \cdot m \cdot 2n \cdot n \cdot n;$

$e.$ $(ab) \cdot (ab) \cdot (ab) \cdot (ab) \cdot (ab) \cdot (ab);$

$f.$ $-q \cdot q \cdot q \cdot q \cdot q;$

$g.$ $4 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7;$

$h.$ $(p-q) \cdot (p-q) \cdot (p-q);$

3. What digits should be put instead of * to get true equality? How many solutions does each problem have?

$a.$ $(2\,*)^2 = **\,1;$ $b.$ $(3\,*)^2 = ***\,6$ $c.$ $(7\,*)^2 = ***\,5$ $d.$ $(2\,*)^2 = **\,9$

4. Without doing calculations, prove that the following inequalities hold:

Example:

$39^2 < 2000$: $39 < 40,$ $39^2 < 40^2 = 1600;$ $1600 < 2000.$

$a.$ $29^2 < 1000;$ $b.$ $48^2 < 3000;$ $c.$ $42^2 > 1500;$ $d.$ $67^2 > 3500$

5. Evaluate:

$(-3)^2;$ $-3^2;$ $(-3)^3;$ $2^7;$ $(-2)^7;$ $-2^7;$ $(2 \cdot 3)^3;$ $2 \cdot 3^3;$ $\left(\frac{1}{3}\right)^2;$ $\frac{1}{3^2};$