school nova™

Review of Homework 17.

1. Mary has two cats, Omlet and Doughnut. Each morning she gives them a can of cat food. Omelet can eat the whole can of food in 15 minutes and Doughnut can eat it in 10 minutes. How many minutes do they need to eat a can of food if they eat it together?

$$\frac{1}{15} + \frac{1}{10} = \frac{2+3}{30} = \frac{5}{30} = \frac{1}{6}$$

2. This Monday Mary put the food into the cat bowl when only Omelet was in the kitchen. So he was eating the food alone for 5 minutes. Then Doughnut joined Omelet. How long will it take them to eat the rest of the food?

$5x\frac{1}{15} = \frac{5}{15} = \frac{1}{3}$ of food will be eaten so $\frac{2}{3}$ of food is remaining

$$\frac{1}{6}:1 = \frac{2}{3}:x \qquad \frac{1}{6}x = \frac{2}{3} \qquad x = \frac{2}{3}x6 = 4$$

3. A pie recipe calls for 4 eggs, 1.5 cup of sugar, and $\frac{2}{3}$ cup of flour. How much sugar and flour do you need to prepare a dough using 9 eggs?

$$\frac{9}{x} = \frac{4}{1.5} \qquad\qquad\qquad \frac{9}{x} = \frac{4}{2/3}$$

Sugar: $x = \frac{9x1.5}{4} = 3.375$ \qquad Flour: $x = 6:4 = 1.5$

4. There are 21 juice bottles out of which 7 bottles are full, 7 are half-full and the remaining 7 are empty to be divided amongst 3 friends equally. You don't have any measuring device. How will you divide them (both bottles and juice) equally?

5. You are traveling and come to a fork in the road. One path takes you to your destination and the other to your death. Unfortunately, you do not know which is which. Luckily, two people are there to guide you, a knight and a knave. You do not know who is who, but you are allowed to approach one of them and ask a single YES-NO- question. What question do you ask to reveal the correct path?
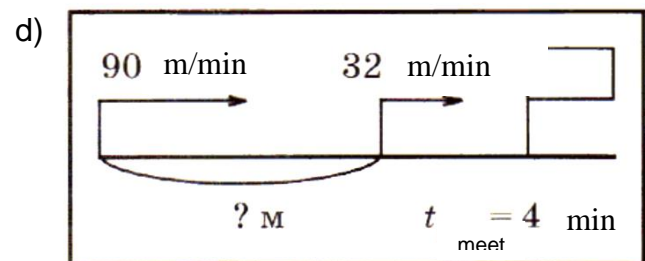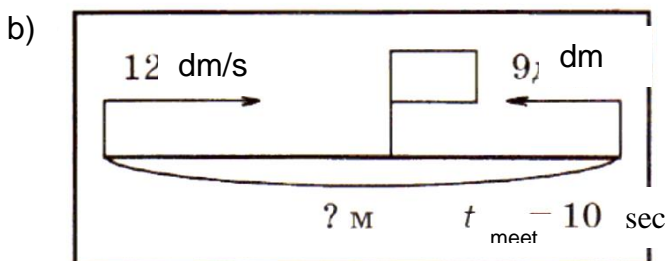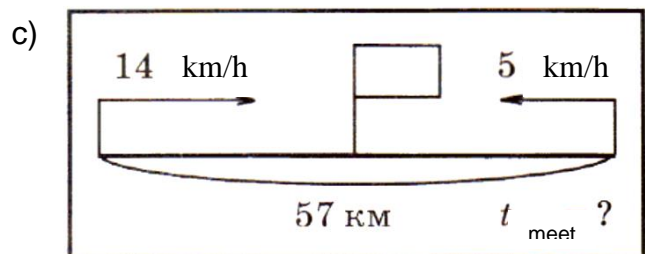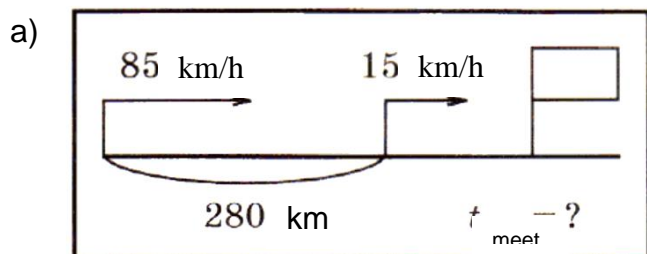
DEATH

6. Two cars start moving at the same time in the same direction from cities A and B, as shown in the picture below.

A   180 km   B
70 km/h   50 km/h

How many hours will it take for the faster car to catch up with the slower car? How far from the city A will they meet?

7. For the four pictures below, come up with the problem and solve it.

a)

85 km/h   15 km/h

280 km $t_{meet} - ?$

b)

12 dm/s   9 dm

? м $t_{meet} - 10$ sec

c)

14 km/h   5 km/h

57 км t_{meet} ?

d)

90 m/min   32 m/min

? м $t_{meet} = 4$ min

Exponents

The main reason we use exponents is because it's a shorter way to write out big numbers.
Exponentiation is a mathematical operation, written as a^n, involving two numbers, the **base** a and the **exponent** n. When n is a positive integer, an exponent tells us to multiply the base by itself that number of times: *We can say that a is raised to the power of n.*

a^n tells you multiply **a** by itself **n** times:

$$\underset{\text{base}}{a}{}^{\overset{\text{exponent}}{n}} = \underbrace{a \times a \times \ldots \times a}_{n}$$

4^3 This tells us to multiply the base 4 by itself 3 times: $4^3 = 4 \times 4 \times 4$

When n is a negative integer, an exponent tells us to divide the by the base that number of times. Or multiply by the base that number of times and take a reciprocal number

$$a^{-n} = \frac{1}{a^n}$$

Properties of exponent:

If the same base raised to the different power and then multiplied:

$$b^3 \times b^4 = (b \times b \times b) \times (b \times b \times b \times b) = b \times b \times b \times b \times b \times b \times b = b^{3+4} = b^7$$

$$\boxed{b^n \times b^m = b^{n+m}}$$

If the base raised to the power of n then raised again to the power of m:

$$(b^2)^3 = (b \cdot b)^3 = (b \cdot b) \cdot (b \cdot b) \cdot (b \cdot b) = b^{2 \cdot 3} = b^6$$

$$\boxed{(b^n)^m = b^{n \cdot m}}$$

$$b^1 = b; \qquad\qquad b^0 = 1, for\ any\ b\ exept\ 0.$$

If two different bases raised to the same power, then:

$$(a \cdot b)^3 = (a \cdot b) \cdot (a \cdot b) \cdot (a \cdot b) = a \cdot a \cdot a \cdot b \cdot b \cdot b = a^3 b^3$$

$$\boxed{(a \cdot b)^n = a^n b^n}$$

1. **Write an expression, using an exponent, that is equivalent to 9×9×9×9×9×9×9**

2. Rewrite the following expression without parenthesis:

$(-1)^0$ $(-1)^{7892222264}$

$(-1)^1$ $(-1)^{7892222265}$

$(-1)^2$ $1^{1000000}$

$(-1)^3$ $(-1)^{1000000}$

3. Write the following expressions as a product or power:

   a. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$;
   b. $2 + 2 + 2 + 2 + 2$;
   c. $a \cdot a \cdot a$;
   d. $a + a + a$;

   e. $\underbrace{x \cdot x \cdot \ldots \cdot x}_{20\ times}$;
   f. $\underbrace{x + x + \cdots + x}_{20\ times}$;

4. Write the following expressions in a shorter way:

   Example: $7 \cdot 7 \cdot 7 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 = 7^3 \cdot 8^4 \cdot 9^5$

   $2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 7 \cdot 7 =$

$$\underbrace{3 \cdot 3 \cdot \ldots \cdot 3}_{n\ times} \cdot \underbrace{5 \cdot 5 \cdot \ldots \cdot 5}_{m\ times} =$$

$$\underbrace{(-4) \cdot (-4) \cdot \ldots \cdot (-4)}_{k\ times} \cdot \underbrace{6 \cdot 6 \cdot \ldots \cdot 6}_{l\ times} =$$

5. Compare using $<$; $>$; $=$

a) $5^2 \quad 2^5$

b) $1^{10} \quad 1^5$

c) $134^1 \quad 250^1$

d) $12^0 \quad 18^0$

e) $3^4 \quad 3^5 x 3^{-1}$

f) $(5x9)^{15} \quad 5^{15} \, x \, 9^{15}$

g) $(-2)^3 \quad -6$

h) $(-2)^3 \quad (-2)^2$

k) $1^{15} \quad 1^{150}$

l) $(-1)^{999} \quad 1^{999}$

m) $2^3 \quad 2^2$

o) $8^{15} \quad 8^{10} \, x \, 8^6$

$=$

6. Solve:

$18 - 2^3 =$

$2^3 \, x \, 2^2 =$

$10^2 =$

$5^2 \, x \, 5 =$

$(2^2)^3 =$

ORDER OF OPERATIONS!!!!!!!

1	2	3	4	5	6
P	**E**	**M**	**D**	**A**	**S**
Parentheses	Exponents	Multiplication	Division	Addition	Subtraction
(....)	a^2	X	÷	+	-