**Fractions:**

> Two fractions are called reciprocal if their product is equal to 1.
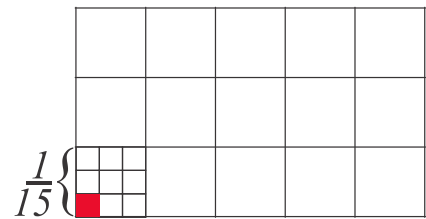> $$\frac{3}{5} \times \frac{5}{3} = 1, \qquad \frac{a}{b} \times \frac{b}{a} = 1$$

Find reciprocal numbers of:

a)  5        b) 1000        c) $\frac{1}{x}$        d) $\frac{13}{17}$        e) -4        f) $-\frac{6}{21}$

If I multiply two fractions (let say $\frac{1}{15}$ by $\frac{1}{9}$),  I will find one ninth part of one fifteenth (or one fifteenth part of one ninth).
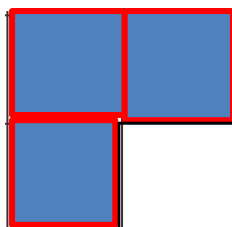
$$\frac{1}{15} \cdot \frac{1}{9} = \frac{1}{15} \div 9 = \frac{1}{15 \cdot 9} = \frac{1}{135}$$
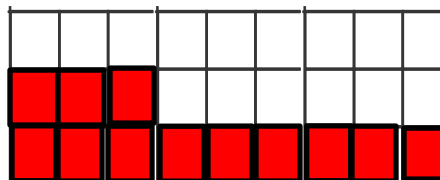
$\frac{1}{15}${

$6: 2 = 3$      $6: \frac{1}{2} =$

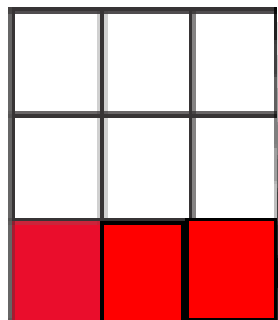**To divide one fraction by another (if they have common denominator) you need to divide only their numerators**

$$\frac{3}{4} : \frac{1}{4} = 3:1 = 3$$

$$\frac{12}{27} : \frac{6}{27} = 12:6 = 2$$

**To divide one fraction by another we should multiply first fraction by the fraction reciprocal to the second fraction.**

$$\frac{3}{9} : \frac{1}{3} = \frac{3}{9} : \frac{3}{9} = 1 = \frac{3}{9} x \frac{9}{3}$$

$$\frac{3}{9} : \frac{1}{18} = \frac{3 \ x \ 18}{9 \ x \ 18} : \frac{9 \ x \ 1}{19 \ x \ 8} = \frac{3 \ x \ 18}{9 \ x \ 1} = \frac{3}{9} \ x \ \frac{18}{1}$$

**Calculate:**

$$\frac{3}{5} : \frac{27}{45} =$$

$$\frac{14a}{48} : \frac{8a}{42} =$$

$$\frac{3}{5} : \frac{11}{5} =$$

$$\frac{9}{10} x \frac{5}{12} =$$

**Calculate:**

9 + (–4) =          9 – (–4) =          –9 – (–4) =          –9 + (–4) =
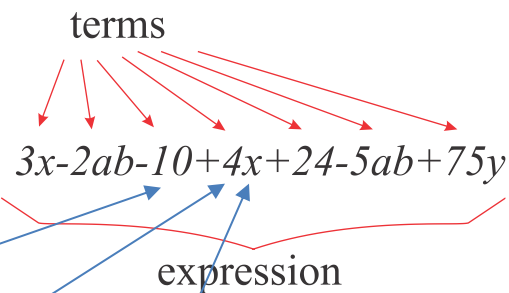
6 + (–8) =          6 – (–8) =          –6 – (–8) =          –6 + (–8) =

**Expressions and equations**

Mathematical expressions - phrases that contain numbers, symbols, letters also known as terms

terms

$3x$-$2ab$-$10$+$4x$+$24$-$5ab$+$75y$

expression

Terms can be:

- Numbers -"constants".
- Letters -"variables"
- Numbers - ("coefficients") combined with ( "variables")

The "like terms" in the expression above are ones that have the same variable.

All constants are like terms

To simplify the expression all like terms should be combined

$$3x - 2ab - 10 + 4x + 24 - 5ab + 75y = 3x + 4x - 2ab - 5ab + 75y - 10 + 24 = 7x - 7ab + 75y + 14$$

- **Simplify:**

a) $1 \text{ cm} + 3 \text{ cm} + 5 \text{ cm} =$

b) $3x + x + 9x - 12x =$

c) $3x + 3 - 5x + 7 =$

d) $3 + x + 2 - 4x =$

- **Remove parentheses:**

a) $2(2x - 1 + 4b) =$

b) $(x + 7 - 5w) \cdot 4 =$

**Equalities**: Is there any difference between two following equalities?

$$a(b + c) = ab + ac \qquad \text{and} \qquad a + 2 = 6$$

Both have letters which are called *variables*.

The first equality will be a true expression for any $a$, $b$, and $c$. This equality describes the distributive property of multiplication over addition.

The second equality will be a true expression for only one value of $a = 4$. This kind of expressions is called an equation.

**How to solve an equation?**

An equation is a statement that the values of two mathematical expressions are equal (indicated by the sign =). *An equation will always contain an equal sign with an expression on each side.*

$$3x - 5 = 4x - 7$$

Solving the equation, means to find such number $x$ that will make the equality true.

First, we must combine all like terms of the expressions.

To keep the equality balanced, equal terms should be added to or subtracted from both sides

Or the term can be moved to another side of the equation with the opposite sign.

$$3x - 5 = 4x - 7$$
$$3x - 3x - 5 = 4x - 3x - 7$$
$$-5 = x - 7$$
$$-5 + 7 = x - 7 + 7$$
$$2 = x$$

Both sides of the equation can be divided (or multiplied) by the same number (or term) and as the result we will get the equality again.

$$4 \cdot (x + 5) = 12$$

$$\frac{4 \cdot (x + 5)}{4} = \frac{12}{4}$$

$$x + 5 = 3$$

$$x + 5 - 5 = 3 - 5$$

$$x = -2$$

# Properties of Equalities

| | |
|---|---|
| **Addition Property** | If $a = b$, then $a + c = b + c$. |
| **Subtraction Property** | If $a = b$, then $a - c = b - c$. |
| **Multiplication Property** | If $a = b$, then $a \cdot c = b \cdot c$. |
| **Division Property** | If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$. |
| **Reflexive Property** | $a = a$ |
| **Symmetric Property** | If $a = b$, then $b = a$. |
| **Transitive Property** | If $a = b$ and $b = c$, then $a = c$. |
| **Substitution Property** | If $a = b$, then $b$ can replace $a$ in any expression. |

**Simplify expression, so that there is 0 (zero) on the right (move everything to the left)**

       a. $2x + a - 7 = x - a + 1$

       b. $3y - b + 4 = b - y - 1$

- **Solve the equations:**

$$3y + \frac{1}{2} = y + \frac{3}{2}$$

| $x + 4 = -1$ | $5 - x = -3$ | $x - (-4) = 0$ |
|---|---|---|
| $3x = 27$ | $40 \div x = 8$ | $z \div 6 = 42$ |

**Homework review:**

You need to cut $\frac{1}{2}$ m from a rope $\frac{2}{3}$ m long. You don't have any tools to do the measurements. How you can do it?

Which part of your original $\frac{2}{3}m$ rope would you need to cut?

In other words what fraction is $\frac{1}{2}$ $of$ $\frac{2}{3}$?

$\frac{1}{2} : \frac{2}{3} = \frac{1}{2} x \frac{3}{2} = \frac{3}{4}$    You will need to cut $\frac{3}{4}$ of the $\frac{2}{3}m$ rope

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There are 80 penguins in a zoo and $\frac{3}{4}$ of them love tuna. While 47 penguins love red tuna, only 42 love yellow tuna. How many penguins love both kinds of tuna?
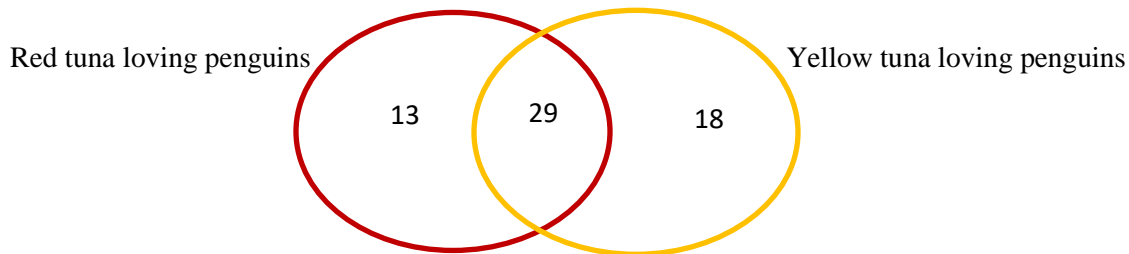
How many penguins love tuna in all? $\frac{3}{4}$ x80=60

60-47=13 (13 do not love yellow tuna)
60-42=18 (18 do not love red tuna)
13+18=31 (31 penguins love either red or yellow tuna)
60-31=29 (29 penguins love both types of tuna)

Red tuna loving penguins                              Yellow tuna loving penguins

13          29          18

Short solution: 47+42=89     and then     89-60=29