Math 4a. Class work 25.

Example 1.

BELLEVILLE

Prix Fixe Menu

[$25]

A weekly prix-fixe menu offering Chef Fabian Pauta's latest culinary creations - using only the finest local ingredients available. This menu is available with or without local wine pairings for each course [$15].

1ST COURSE

Soup of the Day

or

Escargots

½ dozen snails baked in garlic herb butter

or

Grilled Eggplant Salad

goat cheese, baby arugula, tomato, balsamic

or

Tarte Flambée

Alsatian pizza with Prosciutto di Parma, sautéed onions, baby arugula, shaved Gruyère

2ND COURSE

Sautéed Skate

sautéed spinach, marcona almonds, lemon-herb vinaigrette

or

Crispy Pork Belly

caramelized onions, apple compote, arugula and frisée salad, maple-pork jus

or

Coq au Vin

free-range chicken, marinated in Burgundy wine and braised, salt pork lardons, pearl onions and potato purée

or

Le Végétarien

potato gnocchi with roasted tomatoes, black olives, spinach, shaved Gruyère

3RD COURSE

Chocolate Mousse

Raspberry coulis and caramel sauce

or

Lemon Custard

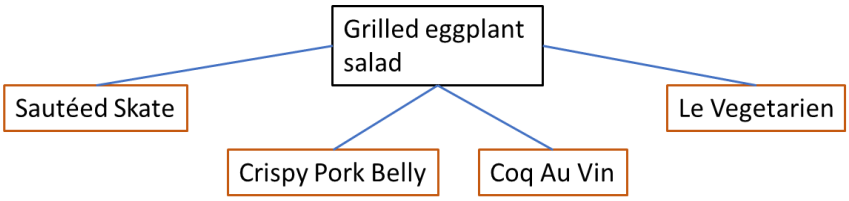
Meyer lemon, sweetened cream, lemon confit

or

Cheese Plate [$5 Supplement]

Peter wants to invite his friends to this restaurant and treat everybody to the fixed-price dinner. What is the maximum number of friends he can invite if he wants every friend to have a different set of dishes? How can the number of possible dinner sets be calculated? Of course, each friend can have only one dish from the list of each course.

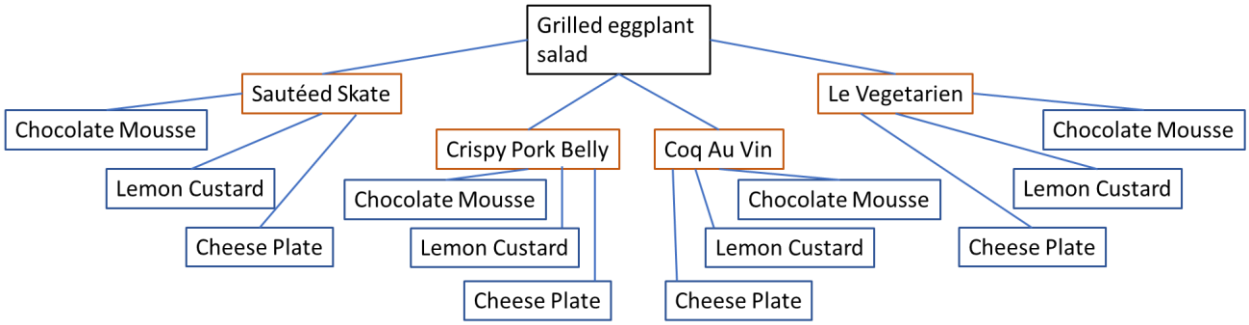
For example, if "Grilled eggplant salad" was chosen as a first course, there are four possible choices for the second course, so four different 1st + 2nd sets of plates can be created:



For every such set of $1st + 2nd$ courses, there are three possible desserts. So,

$$4 \cdot 3 = 12$$

possible entrée and dessert sets with "Grilled eggplant salad" chosen as the appetizer.



There are three more appetizers' choices, four altogether, and for each of them, chosen as an appetizer, there are 12 possible sets of entrée and desserts.

Total number possible different dinner set is $4 \cdot 4 \cdot 3 = 4 \cdot 12 = 48$

Therefor, Peter can invite 48 friends to treat them with dinner, in a way that each has different sat of plates. If there will be more guests, some will eat same dinner set.

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$$4 \cdot 4 \cdot 3 = 4 \cdot 12 = 48.$$

Therefore, Peter can invite 48 friends to treat them to dinner, in a way that each has a different set of plates. If there are more guests, some will eat the same dinner set.

Example 2.

How many different phone numbers exist? In the United States, phone numbers are represented as a 10-digit number, with the first 3 digits as the area code, the next 3 digits as the central office code, and the last 4 digits as the line number.

$$\underbrace{NMX}_{area\ code}\ \underbrace{NXX}_{central\ office\ code}\ \underbrace{XXXX}_{line\ number}$$

Area codes can't start with 0, and 1, middle digit can't be 9.

Central office codes can't start with 0 or 1.

Line number can have any of the 10 digits in any place.

Total number of possible phone numbers is

8 possibilities for 1st digit of the area code, 9 possible digits for the 2nd digit, 10 possibilities for the 3rd.

8 possible digits for the 1st digit of the central office code, 10 possibilities for the 2nd and 10 for the 3rd.

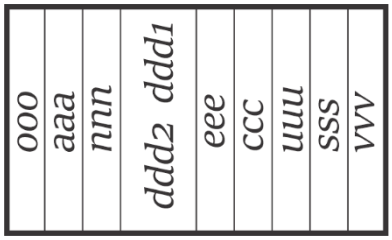
Line number can use any of the digits.

$$8 \cdot 9 \cdot 10 \cdot 8 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 8^2 \cdot 9 \cdot 10^7 = 576 \cdot 10^7$$

Example 3.

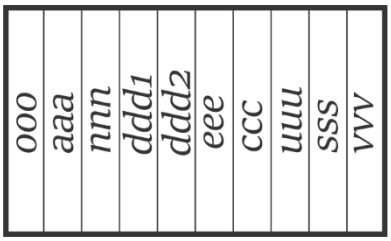
There are 10 books on a shelf. 8 books are written by different authors, and two books are written by the same author. How many different ways are there to put

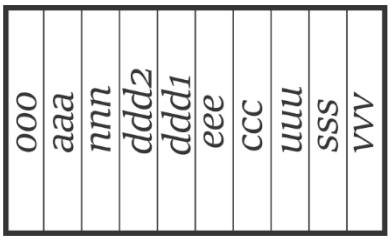
these books on the shelf, so, that two books of the same author are placed next to each other.



Let's imaging those two books of the same author is actually one book. So, we have to rearrange only 9 books. For the first book there are 9 possible places, for the second book there are 8 possible places and so on.

There are $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 9!$ possible ways to rearrange 9 books. But for each such rearrangement, there are two possibilities for the book of the same author to be placed:





Final result is:

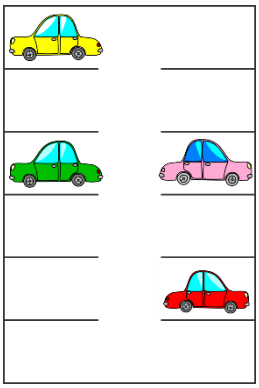
$$9! \cdot 2;$$

Exercises:

1. Mother has 2 apples and 3 pears. Each day she gives one fruit to her kid for lunch. How many different orders are there to give these fruits?

2. Peter took 5 exams at the end of the year. Grades for exams are A, B, C, D. How many different ways are there to fill his report card?

3. Appartment bulding has 12 appartments and a parking for 12 cars (each family has different car). How many different way are there to park these 12 cars?



4. Today there were only 4 cars at the parking lot. How many different ways are there to park 4 cars on a 12-place parking lot?