Math 4a. Class work 24.

There are 5 chairs and 5 kids in the room. In how many ways can the kids sit on these chairs? The first kid can choose any chair. The second kid can choose any of the 4 remaining chairs, the third child has a choice between the three chairs, and so on. Therefore, there are

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ ways how all of them can choose their places. This expression can be represented as $5!$. By definition:

$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5! \qquad or \quad n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1 = n!$$

How many different ways are there to choose 3 students to participate in the math Olympiad, essay competition, and history competition out of 8 students (one student in each competition)?

How many different ways are there to choose a team of 3 students out of 8 to participate in the math Olympiad?

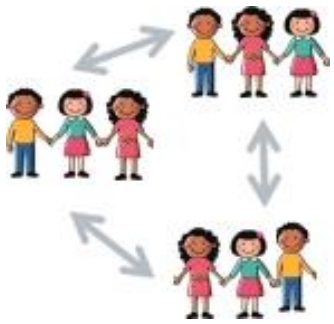
In both cases, we have 8 possible choices for the first student, 7 possible choices for the second student, and 6 different choices for the third one. So, there are $8 \cdot 7 \cdot 6$ different possible groups of students?

What are the similarities in these two problems? Can you see the difference between them?

For example, we can choose Maria to participate in the math Olympiad, Mike to go to the essay competition, and Jessika to participate in History competition. In another possibility Mike will go to the history competition, Jessika to the math Olympiad, and Maria will write an essay.

$$8 \cdot 7 \cdot 6 = 336$$

In the second problem, team consisting of Mike, Jessika, and Maria will be the same regardless of the order they are chosen. So, $8 \cdot 7 \cdot 6$ is $3! = 6$ times greater than the number of possible ways.



1. Maria, Mike, Jessika
2. Maria, Jessika, Mike
3. Mike, Maria, Jessika
4. Mike, Jessi7ka, Maria
5. Jessika, Mike, Maria
6. Jessika, Maria, Mike

Answer for our problem is

$$\frac{8 \cdot 7 \cdot 6}{3 \cdot 2} = 8 \cdot 7 = 56$$

Exercises:

1. Simplify the following fractions:

$\frac{5!}{7!}$;

$\frac{n!}{(n-2)!}$;

2. How many different ways are there to put 64 books on the shelf?

3. There are 20 students in the 4th grade math team. They have to choose 4 participants to go to the county math Olympiad. How many ways are there to choose these 4 students from the team of 20?

4. In the restaurant, there are 3 choices of starters, 4 choices of entrees and 5 choices of tasty desserts in the fix price dinner menu. How many different ways are there to fix a dinner for the restaurant's clients?

5. How many two-digit numbers can be composed from digits 1, 2, 3 without repetition of digits?
6. How many two-digit numbers can be composed from digits 1, 2, 3, if repetition is allowed?

7. There are 10 books on the library shelf. 8 of them are authored by different authors and 2 are from the same author. How many different ways are there to place all these books on a shelf so that 2 books of one author will be next to each other?

8. Mother has 2 apples and 3 pears. Each day she gives one fruit to her kid for lunch. How many different orders are there to give these fruits?

9. Peter took 5 exams at the end of the year. Grade for exams are A, B, C, D. How many different ways are there to fill his report card?

10. There are red and green pencils in a box. How many pencils do you have to take out of the box without seeing them to be sure that you have at least 2 pencils of the same color?

11. If there are pencils of 5 different colors in a box, how many pencils do you have to take out to be sure that you have at least 2 of the same color? 3 of the same color?

12. There are 10 pairs of red gloves and 10 pairs of black gloves in a box. How many gloves do you have to take out to be sure that you have a pair of gloves that you can wear?