Math 4a. Classwork 15.

Substitution.

Let's take a look at a very simple equation.

$$|x| = 10$$

The solution to this equation is a number, the absolute value of which is 10. There are two such numbers, 10 and (-10). Thus, this equation has two roots. (The word "root" can be used as a synonym for solution.).

$$|x| + 5 = 10$$

To make the equation a little simpler, we can substitute $|x|$ with m $(|x| = m)$ and solve for m.

$$m + 5 = 10$$

$$m + 5 - 5 = 10 - 5$$

$$m = 5.$$

But the initial variable is x, not m. $|x| = m$, or, as we know, $|x| = 5$. There are two roots, 5 and (-5).

Word problems.

Equations are very useful to solve word problems. In each word problem there is an unknown quantity, and known parameters out of which the equation can be created. For example, let's take a look on the following problem:

There are 27 pencils in two boxes altogether. There are 5 more pencils In one box then in the other. How many pencils are there in each box?

There are two unknown quantities in this problem, the number of pencils in the first box and the number of pencils in the second box. But these two quantities are not independent, one is 5 less than the other. If the number of pencils in one box is denoted as x, number of pencils in the second box will be $x + 5$. And we also know that the total number is 27.

$$x + x + 5 = 27$$

$$2x = 27 - 5 = 22$$

$$x = 22 : 2 = 11$$

Answer: there are 11 pencils in one box, and 16 in the other.

There are candies in box. If each kid will take 4 candies, 19 candies will be left in the box. If each kid will take 5 candies, there will be lacking 2 candies. How many candies are there in the box?

In this problem there are also two unknown quantities, the number of kids, and number of candies in the box. If the number of kids is denoted as x, the number of candies can be calculated in to ways:

First, $5 \cdot x - 2 =$ number of candies in the box

Second, $4 \cdot x + 19 =$ number of candies in the box, so

$$5 \cdot x - 2 = 4 \cdot x + 19$$

$$5x - 4x = 19 + 2$$

$$x = 21$$

The number of kids is 21. The number of candies can be calculated from either expression:

$5 \cdot 21 - 2 = 4 \cdot 21 + 19 = 103$

Answer: there are 103 candies in the box.

There were 624 books in two boxes altogether. When $\frac{1}{3}$ *of the books from one box and* $\frac{3}{7}$ *of the books from another box were sold to the customers, the number of books in each box became equal. How many books there were in each box at the beginning?*

In this problem there are two unknown variables, number of books in each box. Let's denote the number of books in the first box as x, and the number of books in the second box as y. Together $x + y = 624$. But we know also that

$$\frac{2}{3}x = \frac{4}{7}y$$

$$x = \frac{4}{7}y \cdot \frac{3}{2} = \frac{4 \cdot 3}{7 \cdot 2}y = \frac{6}{7}y$$

We can now substitute x in the equation $x + y = 624$ with $\frac{6}{7}y$.

$$\frac{6}{7}y + y = 624$$

$$\frac{13}{7}y = 624$$

$$y = 624 \cdot \frac{7}{13} = 48 \cdot 7 = 336$$

$$x = \frac{6}{7} \cdot 336 = 288$$

Answer: 288 books, and 336 books.

On the lawn grew 35 yellow and white dandelions. After eight whites flew away, and two yellows turned white, there were twice as many yellow dandelions as white ones. How many whites and how many yellow dandelions grew on the lawn at the beginning?

Again, there are two unknown amounts in the problem: number of yellow and number of white dandelions at the beginning, the sum of these two numbers is 35. We can use y and w as variable names for convenience.

$$y + w = 35$$

Which gives us the following relationship:

$$w = 35 - y$$

Also, we know that

$$2 \cdot (w - 8 + 2) = y - 2$$

$$2(w - 6) = y - 2$$

(eight whites are gone and two yellows are now white, and number of yellows now twice as big as number of whites). Using the substitution $w = 35 - y$, the last equation can be rewritten as

$$2(35-y-6)=y-2$$

$$2(29-y)=y-2$$

$$58-2y=y-2$$

$$58+2=y+2y$$

$$3y=60$$

$$y=20, \quad w=35-20=15$$

Answer: at the beginning there were 15 white and 20 yellow dandelions.

Do you have any idea how to solve these problems without writing un equations?

Examples of solving equations:

1.

$$\frac{9+a}{9}=23$$

We can multiply both sides of the equation by 9:

$$\frac{9+a}{9}\cdot 9=23\cdot 9; \quad 9+a=23\cdot 9$$

Now 9 can be subtracted from both sides:

$$9+a-9=23\cdot 9-9; \quad a=(23-1)\cdot 9=22\cdot 9$$

$$a=22\cdot 9=22\cdot 10-22=220-22=198;$$

Check: $(9+198){:}\,9=23,\ 23=23$

2. Another example:

$$\frac{504}{b-18}=72$$

Both sides of equation can be multiplied by $(b-18)$, of cause $b\neq 18$.

$$\frac{504}{(b-18)} \cdot (b-18) = 72 \cdot (b-18)$$

$$\frac{b-18}{b-18} = 1, \text{so} \qquad 504 = 72(b-18);$$

After using the distributive property:

$$504 = 72(b-18) = 72b - 72 \cdot 18$$

$$72b - 72 \cdot 18 = 504$$

$72 \cdot 18$ can be added to both sides.

$$72b - 72 \cdot 18 + 72 \cdot 18 = 504 + 72 \cdot 18$$

$$72b = 504 + 1296 = 1800; \quad 72b = 1800$$

$$b = \frac{1800}{72} = 25$$

3. More equations:

$$140 - (x{:}\,7 + 29) \cdot 4 = 12$$

Minus sign positioned before the product of $(x{:}\,7 + 29)$ and 4, equation can be written as:

$$140 - \left(\left(x \cdot \frac{1}{7} + 29\right) \cdot 4\right) = 12 \text{ or as}$$

$$140 - \left(\left(\frac{x}{7} + 29\right) \cdot 4\right) = 12$$

Let's open inside parenthesis using the distributive property:

$$140 - \left(\frac{4x}{7} + 29 \cdot 4\right) = 12$$

$$140 - \left(\frac{4x}{7} + 116\right) = 12$$

$$140 - \frac{4x}{7} - 116 = 12; \qquad 24 - \frac{4x}{7} = 12$$

After adding $\frac{4x}{7}$ to both parts and subtracting 12 from both parts:

$$24 - \frac{4x}{7} + \frac{4x}{7} - 12 = 12 + \frac{4x}{7} - 12$$

$$12 = \frac{4x}{7}; \qquad 12 \cdot 7 : 4 = \frac{4x}{7} \cdot 7 : 4;$$

$$\frac{12}{4} \cdot 7 = x; \qquad x = 21$$

Exercises:

1. An apple costs x dollars and a pear cost y dollars. Explain the expressions below:

$$x + y, \qquad x - y, \qquad 3x, \qquad 8y, \qquad 3x + 8y, \qquad y : x, \qquad 120 : y$$

2. Can you create a problem which can be solved by an expression, solve the problem with given data:
   Example: $m + n, \quad m = 5, n = 10$ Problem:
   Mary has m candies, Piter has n candies, how many candies they have altogether?
   $$m + n = 5 + 10 = 15$$
   $a.$ $m : 3 \cdot c;$ $\qquad m = 21, \quad c = 4$
   $b.$ $m \cdot c : 3;$ $\qquad m = 21, \quad c = 4$
   $b.$ $k + 3l;$ $\qquad k = 5, \quad l = 2$

3. Write the following as mathematical expression. If this expression is an equation, solve it.

   a. Sum of the number x and 15 equals to 20.

   b. Product of y and10.

   c. Difference between three times z and 4 is equal to12.

   d. Half of the number b is equal to 1.5

   e. Product of the numbers of 5 and x is less than 12.

4. Can following fractions be reducible (n and k are natural numbers)?

$a.\ \dfrac{2n+1}{2};$ $\quad b.\ \dfrac{3n-1}{6};$ $\quad c.\ \dfrac{2n+1}{2k};$ $\quad d.\ \dfrac{2n+1}{2n-1}$