Math 4a. Class work 6.

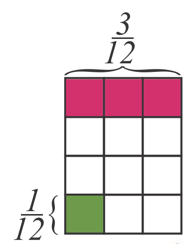
Warm up.

100	200	1000
$24 + \_\_ =$	$\_\_ + 110 =$	$300 + \_\_ =$
$4 \cdot \_\_ =$	$146 + \_\_ =$	$\_\_ \cdot 10 =$
$\_\_ \cdot 10 =$	$\_\_ : 5 =$	$\_\_ + 850 =$
$\_\_ + 89 =$	$1800 : \_\_ =$	$250 \cdot \_\_ =$
$92 + \_\_ =$	$\_\_ + 0 =$	$70 + \_\_ =$

Fractions.

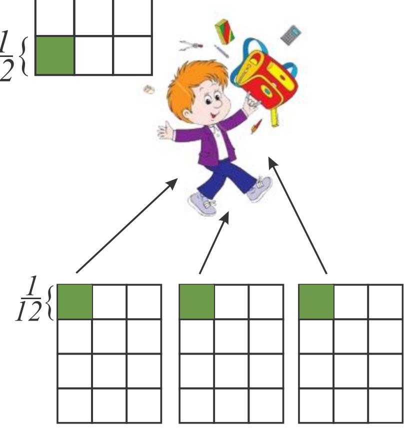
A fraction (from Latin: fractus, "broken") represents a part of a whole.

Look at the picture on the right:



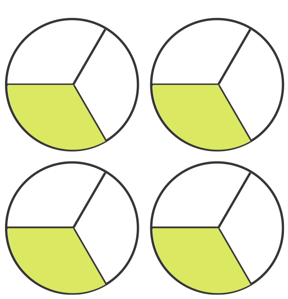
the whole chocolate bar is divided into 12 equal pieces:

$$1\ (whole\ chocolate\ bar): 12(equal\ parts) = \frac{1\ (whole\ chocolate\ bar)}{12(equal\ parts)} = \frac{1}{12}(of\ the\ whole\ chocolate\ bar)$$



To divide 3 chocolate bars among 12 kids we can give each kid $\frac{1}{12}$ of each chocolate bar, altogether

$$\frac{1}{12} + \frac{1}{12} + \frac{1}{12} = 3 \times \frac{1}{12} = \frac{3}{12} = \frac{1}{4} = 3 \div 12$$



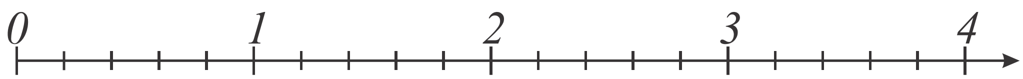
$$3 \div 12 = 3 \times \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$$

To divide 4 pizzas equally between 3 friends we will give each friend $\frac{1}{3}$ of each pizza. Each friend will get $4 \div 3 =$

$4 \times \frac{1}{3} = \frac{4}{3}$, which is exactly 1 whole pizza ($3 \times \frac{1}{3} = \frac{3}{3} = 1$) and $\frac{1}{3}$.

Mark following fractions on the number line:

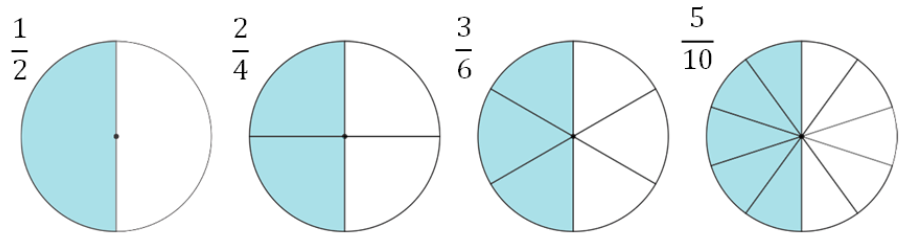
$$\frac{1}{5}, \quad \frac{3}{5}, \quad \frac{3}{3}, \quad \frac{7}{5}, \quad \frac{10}{5}$$



When we talk about fractions, we usually mean the part of a unit. Proper fractions are parts of a unit; improper fractions are the sums of a natural number and a proper fraction. Sometimes we want to find a part of something which is not 1, but can be considered as a single object. For example, among 30 pencils, $\frac{2}{5}$ are yellow. How many yellow pencils are there? What does it mean to find $\frac{2}{5}$ out of 30? The whole pile of all of all these pencils is a single object and we want to calculate how many pencils a little pile of $\frac{2}{5}$ of 30 contains. $\frac{2}{5}$ is 2 times $\frac{1}{5}$, and $\frac{1}{5}$ of 30 is $30 \div 5$. So $\frac{2}{5}$ of 30 pencils will be twice more: $\frac{2}{5} \times 30 = 30 \div 5 \times 2$

Equivalent fractions.

Some fractions can look different, but represent exactly the same part of the whole.



$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{5}{10}; \qquad \frac{1}{2} = \frac{1 \cdot 2}{2 \cdot 2} = \frac{1 \cdot 3}{2 \cdot 3} = \frac{1 \cdot 5}{2 \cdot 5}$$

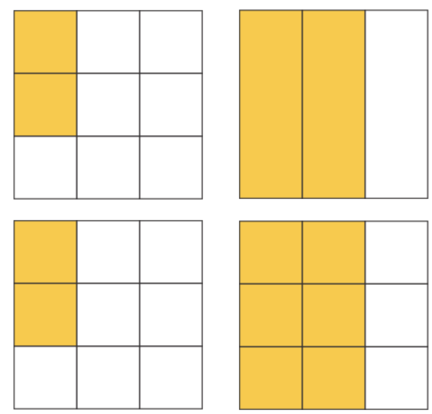
We can multiply the numerator and denominator of a fraction by the same number (not equal to 0), and the fraction will not change; it's still the same part of the whole. We are only dividing the whole into smaller parts and taking more such parts: if parts are twice smaller (denominator is multiplied by 2), we need twice more such parts to keep the fraction the same (numerator is multiplied by 2).

This property of fractions can be used to reduce fractions. If there are common factors in the numerator and denominator, both numbers can be divided by common factors.

$$\frac{25}{35} = \frac{5 \cdot 5}{7 \cdot 5} = \frac{5}{7}; \qquad \frac{77}{352} = \frac{7 \cdot 11}{32 \cdot 11} = \frac{7}{32}$$

Addition and subtraction of fractions with unlike denominators.

Let's try to add $\frac{2}{9}$ and $\frac{2}{3}$. What should we do? Why do we need to bring both fractions to the same denominator? We can add together only similar objects: apples to apples and oranges to oranges. Are two fractions $\frac{2}{9}$ and $\frac{2}{3}$ similar objects?



$$\frac{2}{3} = \frac{1}{3} + \frac{1}{3}, \qquad \frac{2}{9} = \frac{1}{9} + \frac{1}{9}$$

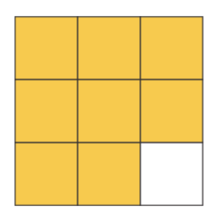
How we can add together

$$\frac{2}{9} + \frac{2}{3} = \frac{1}{9} + \frac{1}{9} + \frac{1}{3} + \frac{1}{3}$$

To be able to add two fractions we have to be sure that they have the same denominator. Each $\frac{1}{3}$ is exactly the same as $\frac{3}{9}$ and $\frac{2}{3} = \frac{6}{9}$

$$\frac{2}{3} \times 1 = \frac{2}{3} \times \frac{3}{3} = \frac{2 \times 3}{3 \times 3} = \frac{6}{9}$$

$$\frac{2}{9} + \frac{2}{3} = \frac{2}{9} + \frac{6}{9} = \frac{8}{9}$$



Mutually prime numbers are numbers which do not have common factors, but 1. Like 8 and 9, they are not prime, but do not have common factors other than 1.

If we multiply both numerator and denominator by the same number, the fraction will not change. Common denominator of both fractions should be the multiple of these denominators. If both numbers are prime (or mutually prime), the least common multiple is their product. If this is not the case, least common multiple is the simplest common denominator, but not the only one, any other multiple can do this task. Nominator and denominator of each fraction should be multiplied by a number to bring both fractions to a common denominator.

For example,

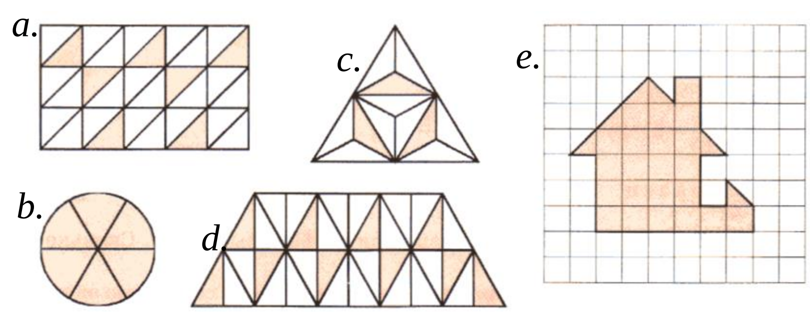
$$\frac{3}{8}+\frac{5}{12}$$

Common denominator can be $8 \cdot 12 = 96$, but 24 is smaller.

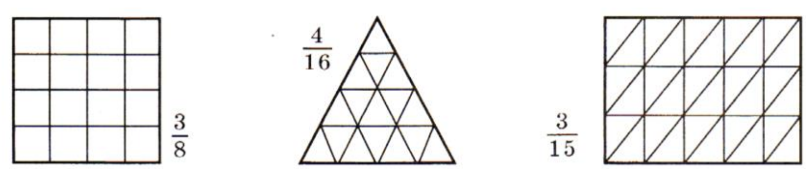
$$\frac{3 \cdot 3}{8 \cdot 3}+\frac{5 \cdot 2}{12 \cdot 2}=\frac{9}{24}+\frac{10}{24}=\frac{19}{24}$$

Exercises.

1. Write a fraction which show the shaded part of the shape:



2. Shade the corresponding part of the figure:

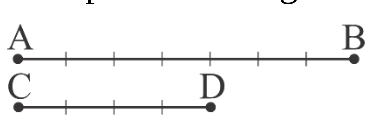


3. Compare:

$$\frac{3}{5} \quad \frac{2}{5} \qquad\qquad \frac{3}{5} \quad \frac{3}{8} \qquad\qquad \frac{3}{6} \quad \frac{1}{2}$$

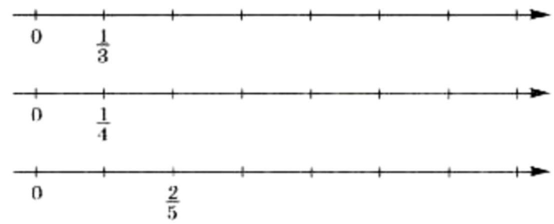
$$\frac{1}{5} \quad \frac{5}{1} \qquad\qquad \frac{4}{12} \quad \frac{3}{4} \qquad\qquad \frac{2}{11} \quad \frac{1}{12}$$

4. What part of the segment [AB] is the segment [CD]?

a. 

b. 

5. On the number lines, mark the number 1.



6. Fill the empty spaces for fractions:

$$\frac{2}{3}=\frac{}{9}=\frac{}{21}=\frac{4}{}=\frac{36}{}$$

7. Calculate:

$a.\ \frac{1}{5}+\frac{1}{2};$ $b.\ \frac{2}{5}+\frac{3}{10};$ $c.\ \frac{5}{9}-\frac{1}{3};$

8.

a. What is bigger, the number *c* or $\frac{2}{3}$ of the number *c*? Why?

b. What is bigger, the number *b* or $\frac{3}{2}$ of the number *b*? Why?

c. What is bigger, $\frac{2}{3}$ of a number *m* or $\frac{3}{2}$ of a number *m*? Why?

9. Simplify fractions:

$\frac{2}{8};$ $\frac{14}{21};$ $\frac{7}{49};$ $\frac{3}{5};$ $\frac{6}{8};$

10.a. $\frac{1}{7}$ of all students in the class is 4. How many students are there in the class?

b. $\frac{2}{5}$ of all students in a class is 10. How many students are there in a class?

11. $\frac{5}{8}$ of a number is 15. What is the number?

12. The kilogram of cookies costs 15 dollars. How much Mary paid for $\frac{4}{5}$ of the kilogram of the cookies.

13. In the school cafeteria there are 12 tables. There are 10 seats at each table. At the lunch time $\frac{4}{5}$ of all sits were occupied by students. How many students were in the cafeteria?

14. An apple worm was eating an apple. On the first day it ate half of the apple, on the second day it ate half of the rest, and on the third day it ate half of the rest again. On the fours day it ate all the leftovers. What part of the apple did it eat on the fourth day?

15. Peter spent 2 hours doing his homework. $\frac{1}{3}$ of this time, he spent doing his math homework and $\frac{1}{4}$ of the remaining time he spent on the history assignment. How many minutes did Peter spent on his history assignment and how many minutes did he spent doing his math homework?

16. Write the expression for the following problems:
   a. 3 packages of cookies cost *a* dollars. How many dollars do 5 of the same packages cost?
   b. 5 bottles of juice cost *b* dollars. How many bottles can one buy with *c* dollars?

17. Half of the students of the class partisipated in a spellingbee competition. One third of them bacame winners. How many students are in the class, if there are 5 winners of the spellingbee in the class?

18. Find

$$a.\ \frac{3}{4}\ of\ 12, \qquad b.\ \frac{2}{7}\ of\ 14, \qquad c.\ \frac{5}{8}\ of\ 56$$

19. There are 48 pencils of each color: blue, yellow and green pencils, 72 red pencils and 120 coloring pictures. How many identical coloring sets can be created out of these pencils and pictures?