Math 4a. Classwork 2.

Last week we discussed a few properties of addition and multiplication. As we all know, multiplication is an arithmetic operation, equivalent to the repetitive addition of the same number.

$$c \times b = \underbrace{c + c + c + \cdots + c}_{b\ times} = \underbrace{b + b + b + \cdots + b}_{c\ times} = a$$

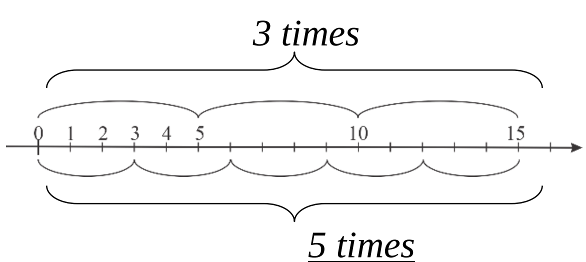
The result of multiplication is called the *product*, and the participants in the operation are called *factors*. c and b are factors, and a is a product.

Multiplication is closely connected with division; when we perform division of a number (this number is called the *dividend*) by a *divisor*, we are seeking a number (a *quotient*) that, when multiplied by the divisor, gives us the dividend.

(In this part of our course, we are discussing natural numbers, which are used for counting and start from 1: 1, 2, 3, and so on. I will omit the word 'natural' and use only the term 'number'.)

If there is a number c, that $c \times b = a$, then we can say that $a \div b = c$. This means that a is divisible by b, and b can be “fit” into a a whole number of times. c is also a factor of a, $a \div c = b$. For example,

$$3 \times 5 = 15; \quad 15 \div 3 = 5, \quad 15 \div 5 = 3$$



$$\overset{\text{factor}}{5} \cdot \overset{\text{factor}}{3} = 5 + 5 + 5 = 3 + 3 + 3 + 3 + 3 = \overset{\text{product}}{15}$$

5 can fit into 15 exactly 3 times, 3 can go into 15 exactly 5 times.
15 is divisible by 3 and by 5.

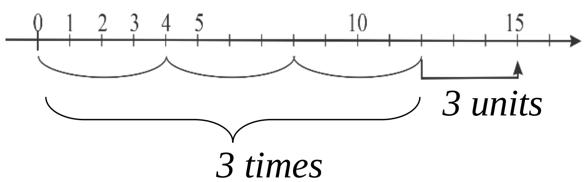
If there is no number such that the divisor enters the dividend several times, then we can say that this number is not divisible by the divisor. In such cases, we can use division with a remainder.

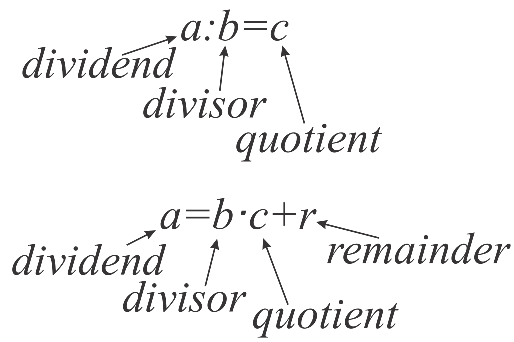
For example, consider $15 \div 4$. 4 can't fully complete 15. It can fit into 12 three times, but there will be a little more left over. So,

$15 \div 4 = 3 \; with \; a \; remainder \; of \; 3$

$15 : 4 = 3R(3)$, or

$$15 = 4 \times 3 + 3$$





For division of any natural number by another, we can now write:

$$a \div b = cR(r), or \; a = b \times c + r$$

If $r = 0$, number *a* is divisible by number *b*.

Why can't we divide by 0? By definition, multiplying 0 by anything results in 0. Dividing by 0 would imply that there is a number that, when multiplied by 0, does not yield 0. But this is impossible. So, Therefore, division by 0 is undefined; it simply does not exist, and we cannot perform such an operation!

Divisibility rules.

Can we predict whether a given number is divisible by 2, 3, 4, and so on? There are divisibility rules:

1. ***Any (natural) number is divisible by 1.***
2. ***A number is divisible by 2 if and only if its last digit is even or 0.***

3. ***A number is divisible by 3 if and only if sum of its digits is divisible by 3.***

4. ***A number is divisible by 4 if and only if the number formed by the last 2 digits is divisible by 4.***

5. ***A number is divisible by 5 if and only if its last digit is 5 or 0.***
6. ***What can you say about the divisibility rule for division by 6? Write it here:***

   ____
   ____
   ____
   ____

7. ***A number is divisible by 7 if and only if the result of subtracting twice the last digit from the remaining part of the number is also divisible by 7.***

8. ***A number is divisible by 8 if and only if the number formed by the last 3 digits is divisible by 8.***

9. ***A number is divisible by 9 if and only if sum of its digits is divisible by 9.***

10. ***What can you say about the divisibility rule for division by 10? Write it here:***

   ____
   ____
   ____
   ____

11. ***Number is divisible by 11 if and only if result of alternation addition and subtraction is divisible by 11.***
   Example: is number 517 divisible by 11?
   $5 - 1 + 7 = 11$. 11 is divisible by 11, so 517 is also divisible.

Exercises.

1. If we want to divide a number by 15, what numbers can we get as a remainder?
2. Do the division, write your answer in a form $a: b = cR(r)$.
   Examples:
   $$25: 4 = 6R(1); \qquad 28: 7 = 4R(0)$$
   $a.\ 36: 5;$ $b.\ 43: 4;$ $c.\ 75: 3;$ $d.\ 126: 5;$ $81: 9;$
3. Evaluate the products and name the factors:
   Example: $3 \cdot 25 = 75, factors\ are\ 3\ and\ 25.$
   $a.\ 4 \cdot 12;$ $b.\ 7 \cdot 11;$ $c.\ 15 \cdot 20;$

4. The remainder of $1932 \div 17$ is 11, the remainder of $261 \div 17$ is 6. Is

$$2193 = 1932 + 261$$

divisible by 17? Is it possible to say without division?

5. Find all natural numbers such that when divided by 5, the quotient and remainder are equal?

6. Factor out the common factor, find the value of the expressions:

   Example:

   $21 + 49 = 3 \times 7 + 7 \times 7 = 7 \times (3 + 7) = 7 \times 10 = 70$

   $a.$ $35 - 25;$ $b.$ $44 + 77;$ $c.$ $81 - 45;$

7. Even or odd number will be the sum and the product of
   a. 2 odd numbers
   b. 2 even numbers
   c. 1 even and 1 odd number
   d. 1 odd and 1 even number

   Can you explain why?

8.
   a. Will the following numbers be divisible by 2:

      123457, 1029384756, 43567219874563157830

   b. by 3

      1347, 45632, 5637984265

   c. by 5:

      5635, 78530, 657932, 45879515

   d. by 7:

      1645, 234, 5478, 889, 16506

9. Write all divisors of numbers: 8, 12, 15, 36

   Example: $D(8)$ *are* $1, 2, 4, 8$

10. Is the product of 1247 and 999 divisible by 3 (no calculations)?

11. Number a is divisible by 5. Is the product $a \cdot b$ divisible by 5?

12. Without calculating, establish whether the product is divisible by a number?

$a.\ \ 508 \cdot 12\ by\ 3$ $\qquad$ $b.\ \ 85 \cdot 3719\ by\ 5$

$c.\ \ 2510 \cdot 74\ by\ 37$ $\qquad$ $d.\ \ 45 \cdot 26 \cdot 36\ by\ 15$

$e.\ \ 210 \cdot 29\ by\ 3, by\ 29$ $\qquad$ $f.\ \ 3800 \cdot 44 \cdot 18\ by\ 11, 100, 9$

13. Without calculating, establish whether the sum is divisible by a number:

$a.\ 25 + 35 + 15 + 45 \quad by\ 5;$ $\qquad$ $b.\ \ 14 + 21 + 63 + 24\ by\ 7$

$c.\ 18 + 36 + 55 + 90 \quad by\ 9;$

14. How many vans are needed to take 55 students on a field trip if a van can take 12 students?

15. The summer vacation is 73 days long. Which day of the week will be last day of vacations if the first day was Tuesday?

16. Show that among any three consecutive natural numbers there will be one divisible by 3.

17. Among four consecutive natural numbers will be a number

   a. Divisible by 2?
   b. Divisible by 3?
   c. Divisible by 4?
   d. Divisible by 5?