Homework 12

Graphical representation of ideal gas processes.

We know that pressure P, volume V and temperature T of the ideal gas are connected with the expression:

$$P \cdot V = n \cdot R \cdot T \tag{1}$$

Which is called "equation of state for an ideal gas". Here n is the number of moles of the gas and R=8.31 J/mole K is the universal gas constant (sometimes it is referred to as Mendeleev-Clapeyron constant).

If we know just to parameters, for example P and V, we can easily calculate the third one – T (if the num,ber of moles is known). So the state of the gas can be conveniently represented graphically – as a point in a coordinate plane with the axes P and V:

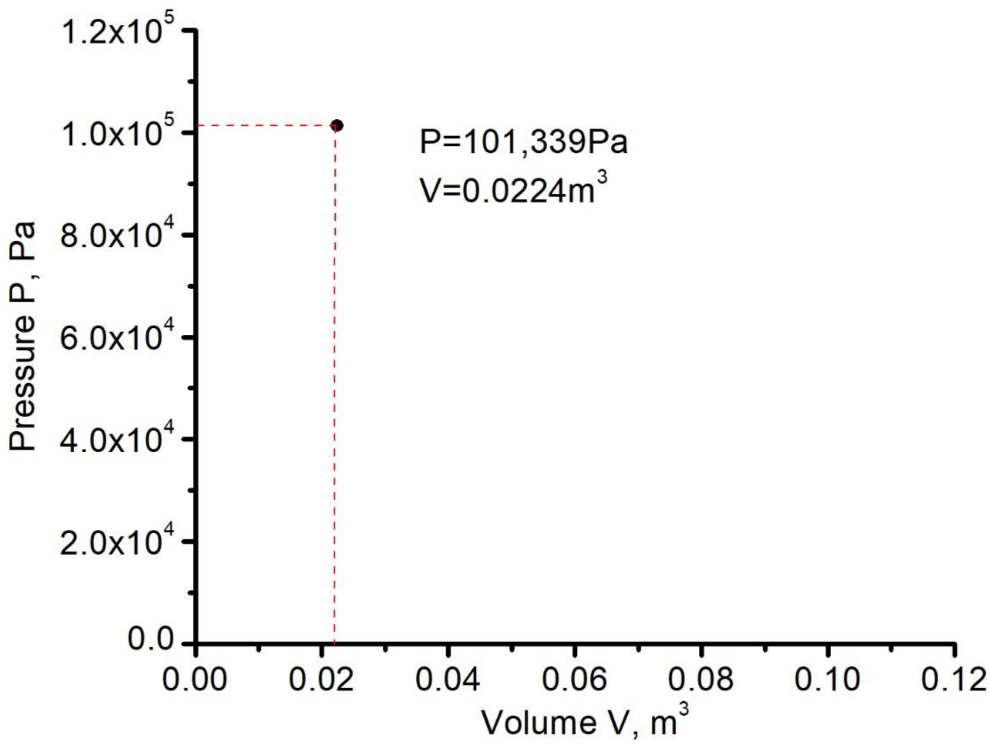

Figure 1. Graphic representation of the state of an ideal gas corresponding to the pressure of 101,339Pa and volume 0,0224m^3.

The black point in Figure 1 corresponds to an ideal gas having pressure P=101,339Pa and volume V=0.0224m^3. If we know the amount of the gas, say, 1 mole, we can easily calculate the temperature using expression (1): T=273.16K.

Now we are reducing the gas pressure to 2x10^4 Pa while keeping volume constant Processes at a constant volume are called *isochoric processes*. This process can be shown as a vertical straight line:

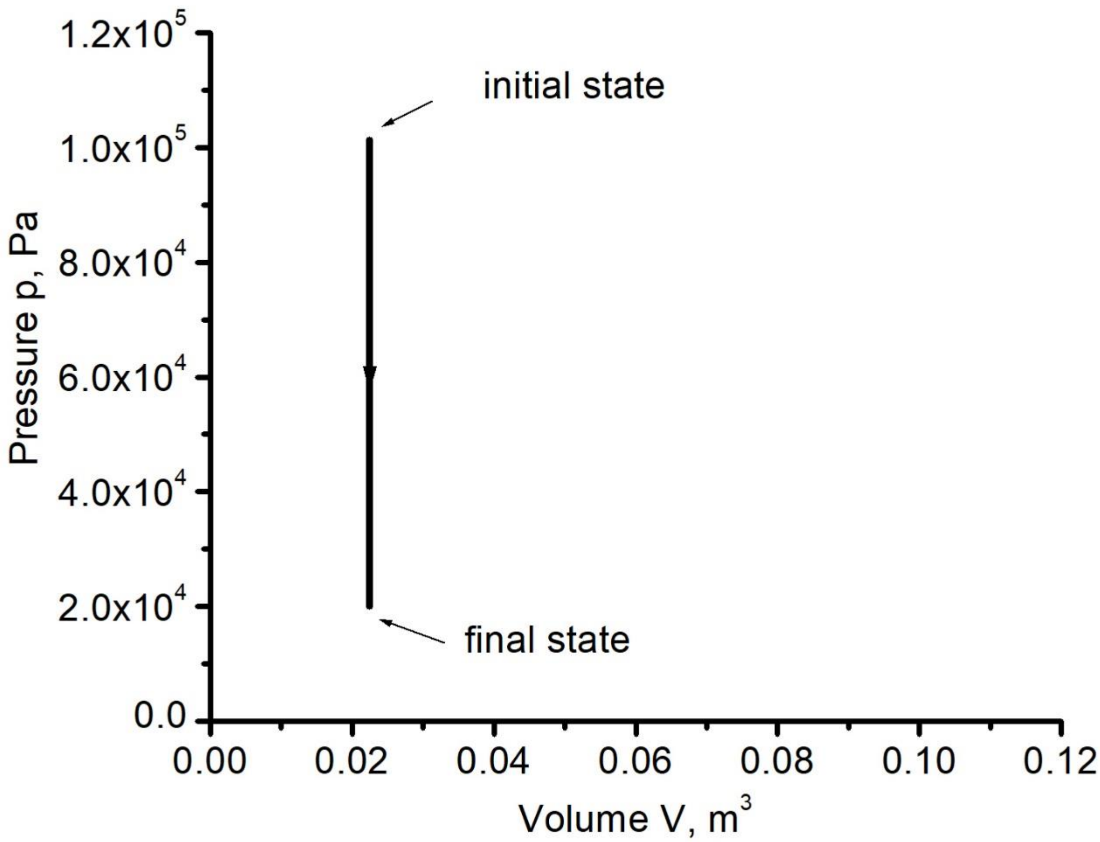

Figure 2. Isochoric pressure decrease.

We can calculate the temperature in the final state using equation 1:

$$T = \frac{P \cdot V}{n \cdot R} = \frac{2 \cdot 10^4 Pa \cdot 0.0224 m^3}{1 \cdot 8.31 \frac{J}{mole \cdot K}} \approx 54K \qquad (2)$$

Another way to calculate the temperature is to use the expression:

$$T_2 = T_1 \cdot \frac{P_2}{P_1} = 273.16K \cdot \frac{20000Pa}{101339Pa} \approx 54K$$

So the temperature decreased during the process. In other words, we just cooled up the gas in a closed volume. Let us assume that the gas does not became liquid at this temperature. For example let us imagine that we are using helium which becomes liquid at 4.2K.

Now, we will increase the volume to 0.113m3 while keeping the pressure constant. Processes at a constant pressure are called *isobaric processes*. The result is shown in Figure 3. Calculation of the temperature gives

$$T = \frac{P \cdot V}{n \cdot R} = \frac{2 \cdot 10^4 Pa \cdot 0.113 m^3}{1 mole \cdot 8.31 \frac{J}{mole \cdot K}} \approx 273.16K \qquad (3)$$

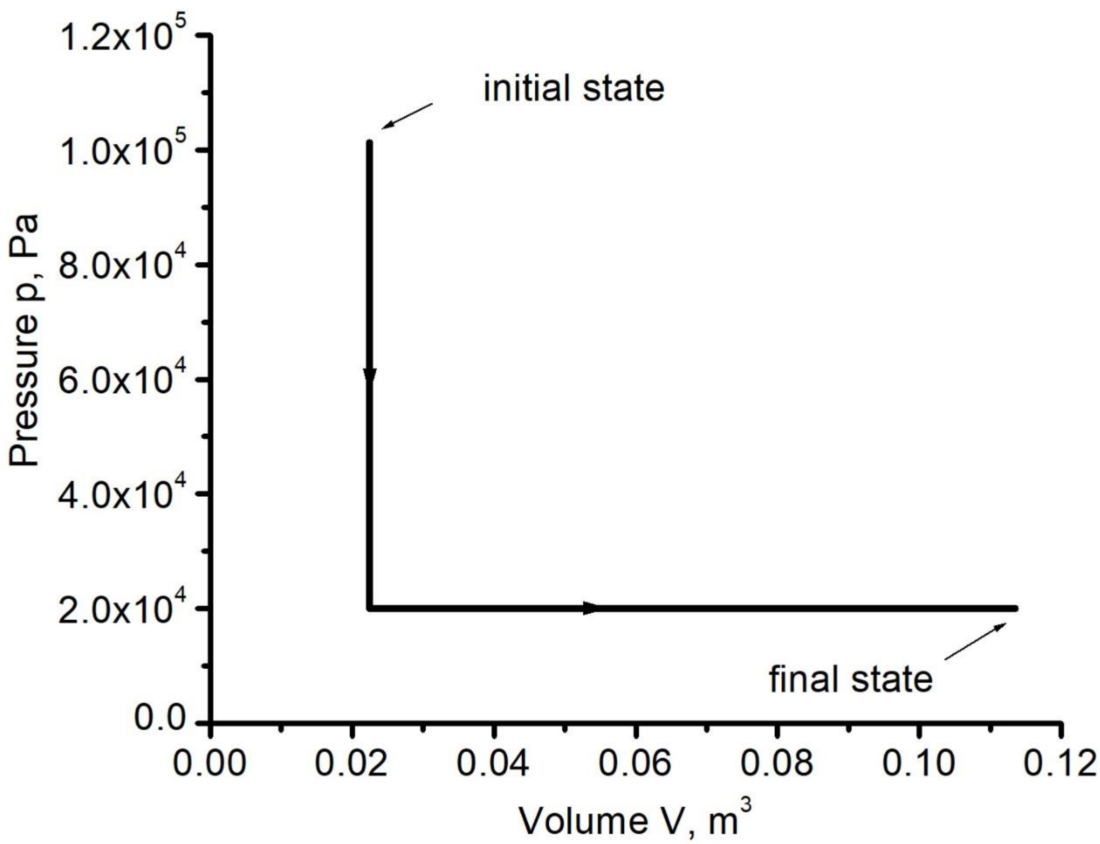

Figure 4. Isochoric pressure decrease and isobaric volume increase.

Now we will decrease volume and increase pressure at the same time while keeping the temperature equal 273.16K. Process at a constant temperature is called *isothermal process*. The dependence of pressure on volume P(V) at a constant T=373.16K can be obtained from the equation of state:

$$P = \frac{n \cdot R \cdot T}{V} = \frac{1\, mole \cdot 8.31 \frac{J}{mole \cdot K} \cdot 273.16K}{V} = \frac{2{,}270J}{V} \quad (4)$$

The result is shown in Figure 5. The expression (4) corresponds to hyperbola (process 1-3).

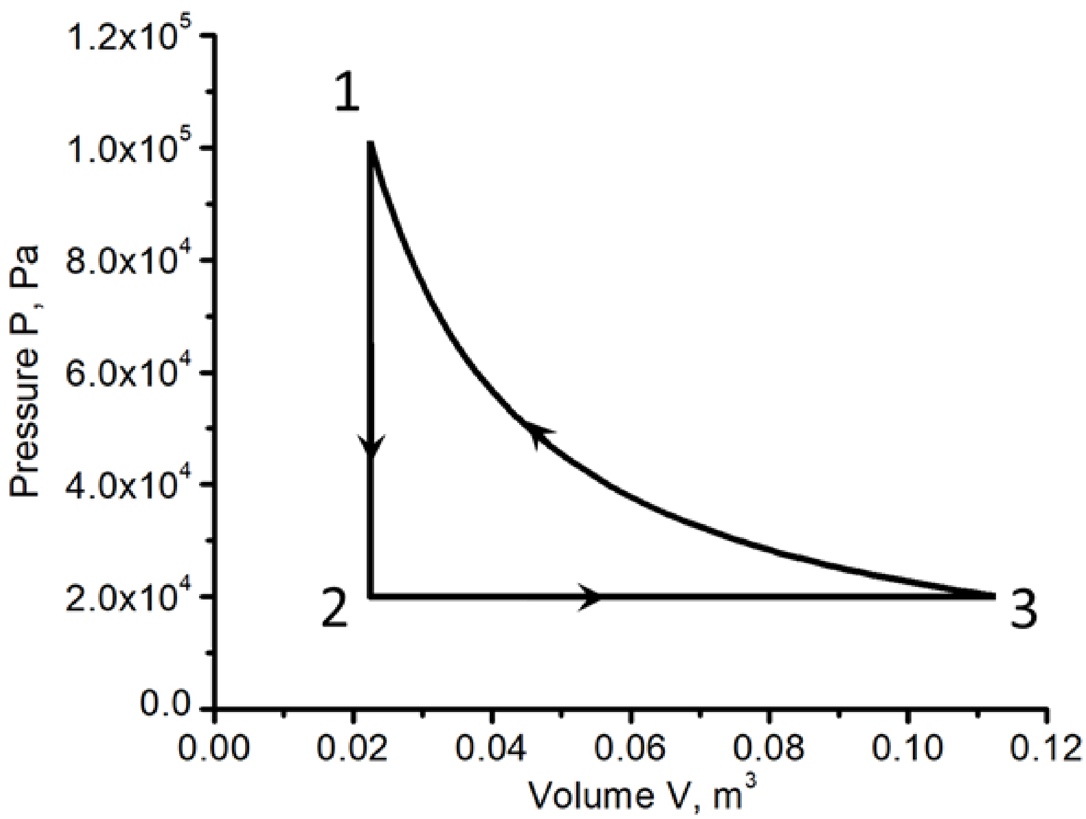

Figure 5. Isochoric (1-2) ,isobaric (2-3) and isothermal (3-1) processes.

Problems:

1. Draw the cyclic process shown in Figure 5 in the coordinates P,T and V,T.
2. There is a cyclic process shown in a Figure below. Show on the graph the points corresponding to the gas states with highest and lowest temperature.