Homework 28: Trigonometry, trigonometric functions.
HW28 is Due May 17; submit to Google Classroom 15 minutes before the class time.

1. Converting radians - degrees

Full circle: $\mathbf{3 6 0}^{\mathbf{0}}=\mathbf{2 \pi} \mathrm{rad}$ and half circle: $\mathbf{1 8 0}^{\mathbf{0}}=\boldsymbol{\pi}$ radians.

- Convert rad to degrees: substitute the $\boldsymbol{\pi}=\mathbf{1 8 0}^{\circ}$ or multiply by $\frac{\mathbf{1 8 0}^{0}}{\boldsymbol{\pi}}$
- Convert degrees to rad: multiply by $\frac{\pi}{180^{0}}$ or use the unit circle.


| Function | Notation | Definition | $0^{0}$ | $30^{0}$ | $45^{0}$ | $60^{0}$ | $90^{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | angle in rad | 0 | $\pi / 6$ | $\pi / 4$ | $\pi / 3$ | $\pi / 2$ |
| $\operatorname{sine}$ | $\sin (\alpha)$ | $\frac{\text { opposite side }}{\text { hypothenuse }}$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| $\operatorname{cosine}$ | $\cos (\alpha)$ | $\frac{\text { adjacent side }}{\text { hypothenuse }}$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 |
| $\tan$ | $\tan (\alpha)$ | $\frac{\text { opposite side }}{\text { adjacent side }}$ | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | $\infty$ |

2. Trigonometric function, $\sin (x)$

By looking at the values of sine as we go around the trigonometric circle, we find that:

- $\sin 0=\sin \pi=0$
- $\sin x$ increases from 0 to $\pi / 2$
- at $x=\pi / 2, \sin x$ reaches its maximum value, 1 .
- at $x=3 \pi / 2, \sin x$ reaches its minimum value, -1 .
- $\sin (x+2 \pi)=\sin x$.

We can see all of the above conclusions in the graph of the function $\sin x$ :


## Homework problems

Instructions: Please always write solutions on a separate sheet of paper. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So please include sufficient explanations, which should be clearly written so that I can read them and follow your arguments.

All angles are measured in degrees or radians.
You can use the Phet simulation link on Google Classroom!

1. Fill out the following table. Make sure you understand how to convert degrees to radians and use the values of sine and cosine that you already know!

| Degrees | Radians | sine | cosine |
| :---: | :---: | :---: | :---: |
| $180^{\circ}$ | $\pi$ | 0 | -1 |
| $45^{\circ}$ |  |  |  |
| $60^{\circ}$ |  |  |  |
| $120^{\circ}$ |  |  |  |
| $150^{\circ}$ |  |  |  |
| $210^{\circ}$ |  |  |  |
| $315^{\circ}$ |  |  |  |
|  | $2 \pi / 3$ |  |  |
|  | $9 \pi / 4$ |  |  |
|  | $5 \pi / 6$ |  |  |
|  | $-5 \pi / 4$ |  |  |
|  | $11 \pi / 3$ |  |  |
|  | $7 \pi / 6$ |  | $\sqrt{3} / 2$ |
|  |  | $\sqrt{2} / 2$ | $-\sqrt{2} / 2$ |
|  |  | $-1 / 2$ | $-\sqrt{3} / 2$ |

2. Submit your graph for $y=\cos (x)$ you did in class (Problem 6.) If you have not done this, graph the function of $\cos (\mathrm{x})$ for x starting at 0 to 360 degrees for all angles as in the table of problem 1.
3. Using the trigonometric circle, show that $\cos x=\sin (x+\pi / 2)$ for any angle $x$. (Hint: draw an angle $x$ in the first quadrant, then add 90 degrees to that angle. Can you find two congruent triangles?)
Then use that $\cos x=\sin (x+\pi / 2)$, and the graph of cosine function to construct (draw) the graph of the sine function, $y=\sin (x)$
