Homework 24: Coordinate geometry, adding graphs.

HW24 is Due April 19; submit it to Google Classroom 15 minutes before the class time.

1. Coordinate geometry: Introduction

The **midpoint** M of a segment AB with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ has coordinates:

$$M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by the following formula:

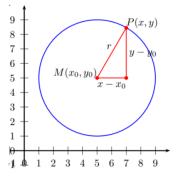
$$d = \sqrt{(x_2 - x_2)^2 + (y_2 - y_1)^2}$$

Line: y = mx + b

with a slope $m=\frac{\varDelta y}{\varDelta x}=\frac{y_2-y_1}{x_2-x_1}$ and intercept **b**.

Parabola: $y = ax^2 + bx + c$ (standard form) or $y = a(x - h)^2 + k$ (vertex form)

Circles: The equation of the circle with the center $M(x_0, y_0)$ and radius r is: $(x - x_0)^2 + (y - y_0)^2 = r^2$.



2. Graphs of functions

In general, the relation between x and y could be more complicated and could be given by some formula of the form y = f(x), where f is some function of x (i.e., some formula which contains x). Then the set of all points whose coordinates satisfy this relation is called the **graph** of f.

- y = f(x) k, shift down by k Vertical shift: y = f(x) + k, shift up by k k > 0
- y = f(x + h), shift to the left by hHorizontal shift: y = f(x - h), shift right by h h > 0
- Reflection, x-axis: y = -f(x), multiply the function by -1.
- Reflection, y-axis: y = f(-x), multiply the argument by -1.

3. Quadratic function (revisited +)

Quadratic equation in a standard form: $ax^2 + bx + c = 0$

- a, b, c coefficients, determinant D: $D = b^2 4ac$, solutions(roots): $x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$ **D** determines the number of roots! (D < 0 no solutions, D = 0 one solution, D > 0 two solutions)

Quadratic function in a factored form: $y = a(x - x_1)(x - x_2)$, where

- roots: the numbers x_1 and x_2 solutions of the quadratic equation (y=0)
- Vieta's formulas: The roots are related to the coefficients: $x_1x_2 = \frac{c}{a}$ and $x_1 + x_2 = -\frac{b}{a}$

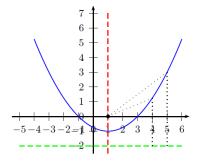
Quadratic function in a vertex form: $y = a(x - h)^2 + k$

- Method 1: completing the square. Use the formulas for fast multiplication.

Method 2: find the vertex. Determine the coefficients
$$a, b, c$$
. Find the vertex x-and y-coordinates $x_v = h = -\frac{b}{2a}$. $y_v = k = y(x_v) = ax_v^2 + bx_v + c$

Modified vertex form: rewrite the equation into separate $y - \text{and } x - \text{part } 4p(y - k) = (x - k)^2$

Distance from any point on the parabola to focus and directrix: $p = \frac{1}{4\pi}$ Vertex V(h, k) Focus F(h, k + p) directrix y = k - p



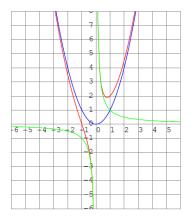
NEW: Parabola is the set of all points in a plane that are equally distant away from a given point and a given line (see black dotted lines). This given point is called the focus (black dot) of the parabola and the line is called the directrix (green line).

If the parabola is of the form $(x - h)^2 = 4p(y - k)$, the vertex is (h, k), the focus is (h, k + p) and the directrix is y = k - p.

4. Adding Graphs

Now that we know how to draw a lot of basic graphs and how to use transformations, we can draw more complicated graphs — that is, graphs that we get by adding two functions. For example, if we want to draw a graph of a function $y = x^2 + \frac{1}{x}$

We can carefully examine two separate graphs of $y = x^2$ (blue) and $y = \frac{1}{x}$ (green), and then see what happens if one adds these two graphs (red) by adding their y-values for every x.



2

Homework problems

Instructions: Please always write solutions on a separate sheet of paper. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer and some justification for why this is indeed the answer. So please include sufficient explanations, which should be clearly written so that I can read them and follow your arguments.

ALL GRAPHS/POINTS/FIGURES SHOULD BE DRAWN BY YOU - NOT PRINTED! **USE QUADRILE PAPER!**

1. Graph $x^2 = 4y$. What is the focus, directrix, and vertex of the parabola?

2. Sketch the following functions by first drawing the graph of each addend function and then adding the y-values for a few x-values. (Review your class notes)

a.
$$y = |x| + |x + 1|$$

b.
$$y = |x - 1| + |x + 1|$$

c.
$$y = |x - 1| - |x + 1|$$

d. |y| = x (Hint: what is the domain/range of this function?)

3. Sketch the following functions by first drawing the graph of each addend function and then adding the y-values for a few x-values.

a.
$$y = x + \frac{1}{|x|}$$

b.
$$y = \sqrt{x} + \frac{1}{x}$$

c. $y = x - \frac{1}{x^2}$

c.
$$y = x - \frac{1}{x^2}$$

4. Find all intersection points of the parabola $y = x^2$ and the circle with a radius $\sqrt{6}$ and center at (0,4).