MATH 6: HANDOUT XVIII
COORDINATE GEOMETRY 3: EQUATION OF A CIRCLE

Equation of a line. Last week, we learned that we can express a line in the coordinate plane through the following linear equation

$$y = mx + b,$$

where m stands for the slope of the line and b stands for the y-intercept.

If we know two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ through which the line passes, we can find the slope m by doing

$$m = \frac{y_1 - y_2}{x_1 - x_2}.$$

Midpoint of a segment: If we have two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, the midpoint of the segment connecting these two points is given by

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Parallel and perpendicular lines: Parallel lines are defined by having the same slope $m_1 = m_2$. In perpendicular lines, the slopes of the two lines are related by $m_1 = -1/m_2$.

Distance between two points: In order to find the distance between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in the coordinate plane, we can make use of the Pythagorean theorem.

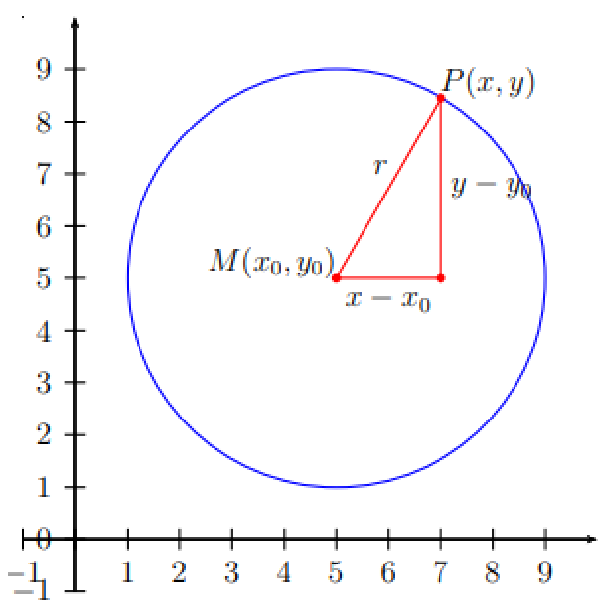
If we connect the two points by a straight line, then we can make a right triangle where this line is the hypotenuse. The legs will then be a horizontal and a vertical line. From this, we can use the Pythagorean theorem to find that the distance d between the two points is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

Equation of a circle: A circle is defined as all the points in the coordinate plane that have equal distance to a fixed point $M(x_0, y_0)$. The distance to this point is what we call the RADIUS of the circle. The equation that describes a circle centered at $M(x_0, y_0)$ with a radius of r is given by

$$(x - x_0)^2 + (y - y_0)^2 = r^2.$$

Can you see any similarity with the equation of the distance between two points?



1. Find the equation of the line through $(1, 1)$ with slope 2.

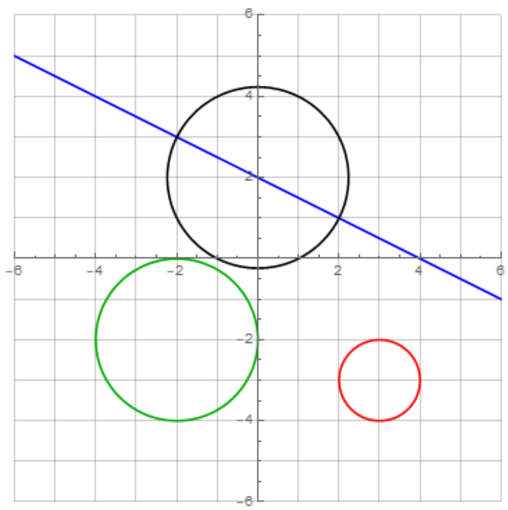
2. Find the equation of the line through points $(1, 1)$ and $(3, 7)$.

3. Consider the following system of linear equations:

$$\begin{cases} 6x - 5y = -3 \\ x + \ \ y = 5 \end{cases}$$

(a) Solve the system of linear equations to find a value for x and y.
(b) Rewrite each of the equations in the form of $y = mx + b$.
(c) Graph each of these lines in a coordinate plane and find their intersection point.
(d) What can you say about the intersection point and the system of linear equations?

4. Find the equation of the circles and the line in the figure below. The black circle has a radius of $\sqrt{5}$.



***5.** Using your answer to the previous question, solve the following system of **non-linear** equations:

$$\begin{cases} y \quad = -x/2 + 2 \\ 5 = x^2 + (y-2)^2 \end{cases}$$

6. (a) Draw the graph of the equation $x^2 + y^2 - 1 = 0$.
(b) Draw the graph of the equation $x^2 + (y-1)^2 - 1 = 0$.
(c) Draw the graph of the equation $(x+2)^2 + (y+3)^2 = 4$.
(d) Draw the graph of the equation $xy = 0$.

7. (a) 3 points $A(0, 0)$, $B(1, 3)$, $D(5, -2)$ are vertices of a parallelogram $ABCD$. What are the coordinates of point C?
(b) 3 points $A(0, 0)$, $B(2, 3)$, $D(4, 1)$ are vertices of a parallelogram $ABCD$. What are the coordinates of point C?
(c) 3 points $A(0, 0)$, $B(1, 5)$, $D(3, -2)$ are vertices of a parallelogram ABCD. What are the coordinates of the point C?
(d) Can you guess the general rule: if $A(0, 0)$, $B(b_1, b_2)$, $D(d_1, d_2)$ are 3 vertices of a parallelogram, what are coordinates of point C?

8. Consider the triangle ΔABC with the vertices $A(-2, -1)$, $B(2, 0)$, $C(2, 1)$. Find the coordinates of the midpoint of B and C. Find the length of the median (i.e. a median unites a vertex with the midpoint of the opposite side) from A in the triangle ΔABC.