MATH 6: HANDOUT XVII
COORDINATE GEOMETRY 2: DISTANCES

Equation of a line. Last week we learned that any relationship between two variables x and y that can be written as

$$y = mx + b$$

will describe a line in the coordinate plane. Here, m represents the slope, and b is the y intercept. The slope tells us how many squares our line goes up (or down) per each square we move to the right. The y intercept tells us where our line crosses the y axis.

Slope from two points. If we know two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ through which the line passes, we can find the slope m by doing

$$m = \frac{y_1 - y_2}{x_1 - x_2}.$$

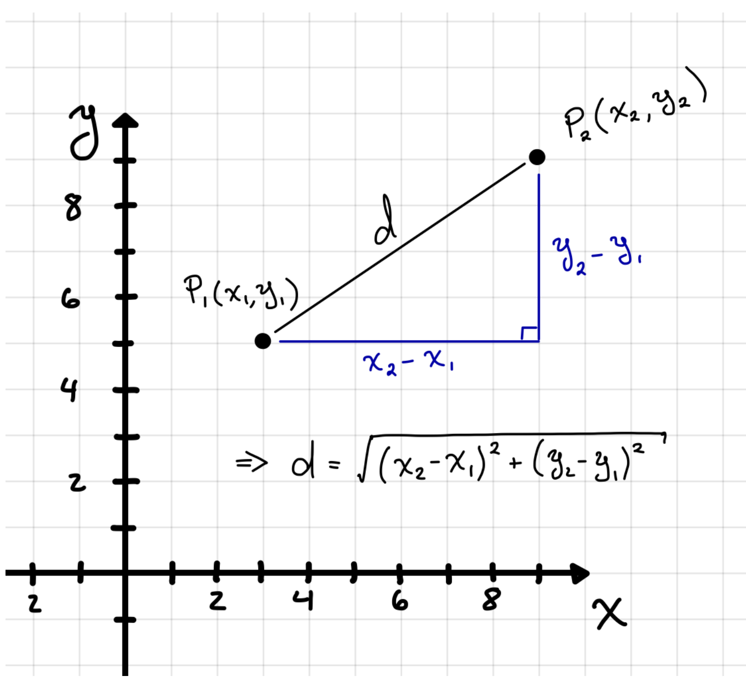
Midpoint of a segment: If we have two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, the midpoint of the segment connecting these two points is given by

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Parallel and perpendicular lines: Parallel lines are defined by having the same slope $m_1 = m_2$. In perpendicular lines, the slopes of the two lines are related by $m_1 = -1/m_2$.

Distance between two points. It is sometimes very useful to be able to determine the distance between any two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in the coordinate plane. The trick is simple: we can always create a right triangle in which the hypotenuse is the line connecting the points and the other sides are purely horizontal and vertical. Therefore, we can use the Pythagorean theorem to find the distance d as

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



1. Given any two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, find a general expression for b.

2. Sketch and find the midpoint and the distance for the following pairs of points:

(a) $A = (3, 11)$ and $B = (7, 5)$
(b) $C = (2, 8)$ and $D = (4, 12)$
(c) $E = (1, 1)$ and $F = (4, 5)$

3. For each of the equations below, draw the graph, then draw the perpendicular line to it (going through the point (0, 0)) and then write the equation of this perpendicular line

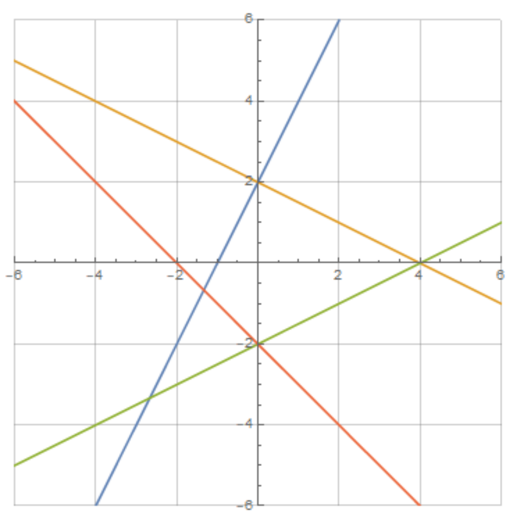
(a) $y = 2x$
(b) $y = -x$
(c) $y = 3x$
(d) $y = -\frac{1}{2}x$

4. Find the equation of a line parallel to each of the graphs from the previous problem that goes through point (2, 0).

5. Let l_1 be the graph of $y = x + 1$, l_2 be the graph of $y = x - 1$, l_3 be the graph of $y = -x + 1$, and l_4 be the graph of $y = -x - 1$.

(a) Find the intersection point of l_1 and l_3; Label this point A and write down its coordinates.
(b) Find the intersection point of l_2 and l_4; Label this point B and write down its coordinates.
(c) Find the midpoint of AB and write down its coordinates.
(d) Let C be the intersection of point l_1 and l_4, and let D be the intersection of l_2 and l_3. What kind of quadrilateral is $ABCD$?
(e) Show that l_1 and l_2 are parallel.

6. Find the equation of the four lines in the figure below. Determine if there are lines that are perpendicular to each other and show why or why not.



7. Find the distance of all the following points to the origin (0,0): $A = (1, 0)$, $B = (-1, 0)$, $C = (-\sqrt{2}/2, \sqrt{2}/2)$, $D = (\sqrt{3}/2, 1/2)$, $E = (\sqrt{2}/2, \sqrt{2}/2)$, $F = (-\sqrt{2}/2, -\sqrt{2}/2)$, $G = (\sqrt{3}/2, -1/2)$. What do all of these points have in common? Try to sketch them and find a visual relationship between all this points.