

MATH 6: HANDOUT XV
FACTORIZATION AND SYSTEMS OF LINEAR EQUATIONS

1. FACTORIZATION

When handling with large algebraic expressions, it is often possible to simplify them. One of doing this is by **factorization**. As its name suggests, this method consists of finding a common factor in two or more terms. For example, in the following expression

$$7x + 9x - 5x$$

the three terms share the common factor x . Therefore, we can rewrite this expression as:

$$7x + 9x - 5x = (7 + 9 - 5)x = 11x.$$

In general, we will have the following identities:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

$$ab + ac = a(b + c)$$

2. SYSTEMS OF LINEAR EQUATIONS

Systems of linear equations are two or more linear equations that are using the same variables, hold true at the same time and have to be solved together. Knowing how to solve systems of equations is useful especially when solving word problems. We are going to learn today how to solve a system of two linear equations using the substitution method and practice solving word problems.

When using the substitution method, select the simpler equation and try to express one variable in terms of the other variable, then substitute the variable in the other equation. You end up with an equation in one variable that you already know how to solve.

Let's look at an example:

$$\begin{cases} 6x - 5y = -3 \\ x + y = 5 \end{cases}$$

Let's choose the second equation $x + y = 5$, $x = 5 - y$. Substitute x with $5 - y$ in the first equation. $6(5 - y) - 5y = -3$, $30 - 6y - 5y = -3$, $11y = 30 + 3$, $y = 3$. To find x , substitute y in the second equation. $x = 5 - 3$, $x = 2$. You can always check that your answer is correct by plugging in 2 for x and 3 for y in the original two equations.

HOMWORK

1. Factor:
 - a. $6a + 12 =$
 - b. $mn + n =$
 - c. $5xy - 15x =$
 - d. $4ax - 8ax^2 + 12ax^3 =$
2. Factor using the factorization identities we learned above:
 - a. $9 - x^2 =$
 - b. $x^6 - 4 =$
 - c. $9 - 6x + x^2 =$
 - d. $a^3 - 2a^2x + ax^2 =$
3. Show that the left hand side (LHS) = right hand side (RHS):
 - a. $(m - n)(a + b) + m - n = a(m - n) + (b + 1)(m - n)$
 - b. $x^2(x + 1) - x - 1 = x(x + 1)^2 - (x + 1)^2$
 - c. $2x(x + b) + a(x + b) = (2x + a)x + (2x + a)b$
 - d. $(a + b)^2 + c(a + b) = (a + b)(a + c) + (a + b)b$
4. Solve the following system of linear equations:
$$\begin{cases} 5x + 2y = 16 \\ 2x + 3y = 13 \end{cases}$$
5. Solve the following system of linear equations:
$$\begin{cases} \frac{5}{6}x - \frac{9}{10}y = -2 \\ \frac{1}{3}x + \frac{2}{5}y = 3 \end{cases}$$
6. Solve the following system of linear equations:
$$\begin{aligned} 2(x + 1) + 3(y - 1) &= 9 \\ 3(x + 2) - 2(y + 1) &= 6 \end{aligned}$$
7. The sum of two numbers is 27. Twice the larger number is 11 less than 3 times the smaller number. What are the two numbers?
8. The sum of two numbers is $\frac{41}{35}$ and the difference is $\frac{1}{35}$. What are the two numbers?
9. A 2-digit number is larger by 9 than the number with the digits reversed. The sum of the digits is 7. Find the number. **Hint:** One can always write $54 = 5 \times 10 + 4$.
- *10. A motor boat can travel 45 miles downstream in 3 hours and 22 miles upstream in 2 hours. Find is the speed of the boat in still water and find the speed of the current. (Hint: speed=distance/time)
- *11. A tank can be filled in 10 minutes from faucet A at a rate of 50ml/s. If another faucet B is turned on when the tank is one-third full, it will take another 4 minutes and 10 seconds to fill the tank. Find the flow of water from faucet B.