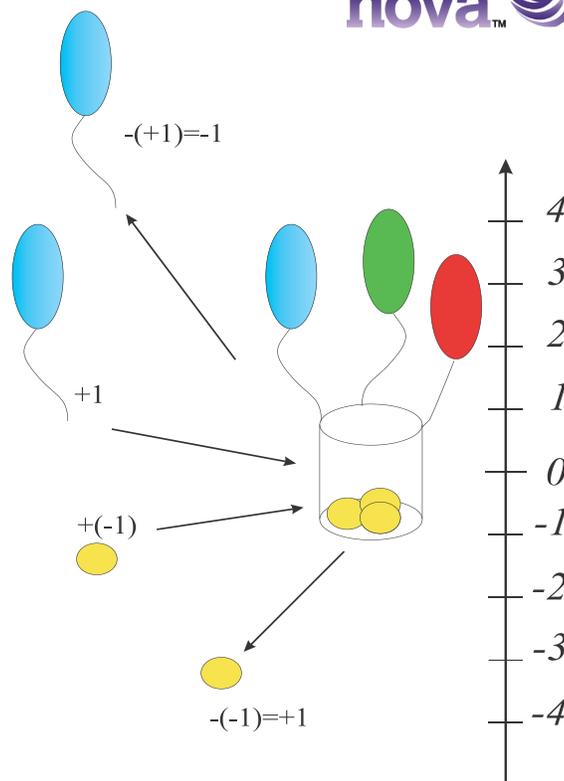
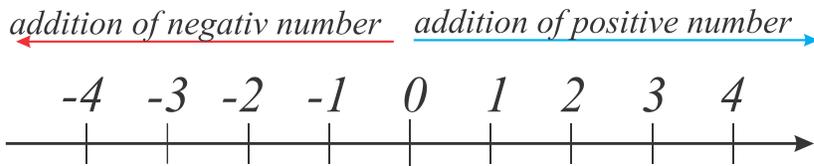


### Positive and negative numbers.

Negative numbers represent opposites. If “positive” represents movement to the right, “negative” represents movement to the left. If “positive” represents above sea level, then “negative” represents below level. If “positive” represents a deposit, “negative” represents a withdrawal. They are often used to represent the magnitude of a loss or deficiency. Negative numbers appeared for the first time in history in the Nine Chapters on the Mathematical Art, which in its present form dates from the period of the Chinese Han Dynasty (202 BC – AD 220), but may well contain much older material. Liu Hui (c. 3rd century) established rules for adding and subtracting negative numbers. By the 7th century, Indian mathematicians such as Brahmagupta were describing the use of negative numbers. Islamic mathematicians further developed the rules of subtracting and multiplying negative numbers and solved problems with negative coefficients. Western mathematicians accepted the idea of negative numbers by the 17th century. Prior to the concept of negative numbers, mathematicians such as Diophantus considered negative solutions to problems “false” and equations requiring negative solutions were described as absurd



We can use the very clear illustration of how positive and negative number work. At the beginning basket has a few balloons attached and the same number of sand bags (weights), placed at 0 position and doesn't move. Balloons represent positive units, sand bags represent negative units. If we add one balloon the basket gets pulled up by one unit. If we add one sand bag, the basket will be pulled down by one unit. If we remove one balloon, basket will go one unit down, which is equivalent of adding one sand bag. So  $-(+1) = +(-1)$ . If we remove one sand bag, basket will go one unit up, which is equivalent of adding one balloon. So  $-(-1) = +( +1)$ .



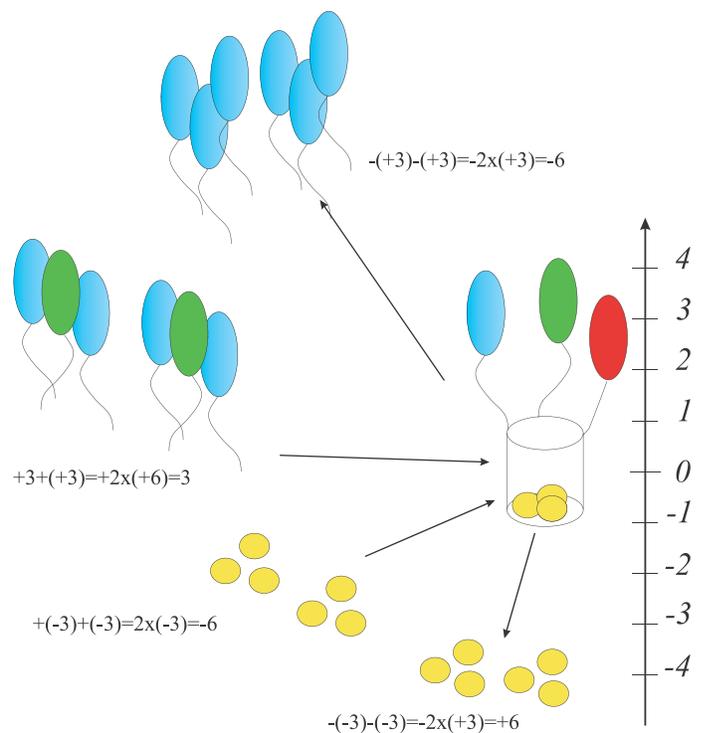
Two numbers that have the same magnitude but are opposite in signs are called opposite numbers.

**Multiplication and division of negative numbers.**

If we multiply 2 positive numbers, we will get third positive number. What will happened if we multiply one negative and one positive number. Let's again review our model. In this case we will add or remove our balloons and sand bags by groups of three. Addition of two groups of 3 sand bags will drive the basket 6 units down, because we add 6 bags. So  $2 \times (-3) = -6$ .

Removing of 2 groups of 3 sand bags will drive our basket 6 units up, which is corresponding of adding 6 balloons, so  $-2 \times (-3) = 6$

Addition of 6 balloons (2 groups of three) of cause will help us to move our basket up for 6 units. If we remove 2 groups of 3 balloons, we will descend 6 units.  $-2 \times (+3) = -6$ .



**Exercises:**

1. Fill up the table:

<i>a</i>	7	-4			5		0	
<i>-a</i>			0	-1		8		-3

2. What number is

$$-(+5); \quad +(+5); \quad -(+5); \quad -(-(+5)); \quad -(-(-5)); \quad -(-(-(-5)))$$

3. Rewrite without parenthesis:

Example:

$$30 - (2 + 1) = 30 - 2 - 1, \quad 30 - (2 - 1) = 30 - 2 + 1$$

To check your solution, you can find the value for both part of the equality:

$$30 - (2 - 1) \neq 30 - 2 - 1$$

$$30 - (2 - 1) = 30 - 1 = 29; \quad 30 - 2 - 1 = 27.$$

$$30 - (2 - 1) = 30 - 2 + 1;$$

$$30 - (2 - 1) = 30 - 1 = 29, \quad 30 - 2 + 1 = 29$$

a.  $20 + (2 - 3);$

b.  $20 - (2 - 3);$

c.  $20 - (-2 + 3);$

d.  $20 - (-2 + (-3));$

4. Compare:

$$-4 \quad 4$$

$$6 \quad -4$$

$$\frac{2}{3} \quad -\frac{3}{2}$$

$$-4 \quad -2$$

$$-4 \quad 0$$

$$-\frac{2}{3} \quad -1$$

$$-4 \quad -6$$

$$-1 \quad -\frac{1}{2}$$

$$-2 \quad \frac{1}{2}$$

5. Evaluate:

$$3 + (-2);$$

$$3 + (+2);$$

$$-3 - (-2);$$

$$3 - (+2);$$

$$-3 + (-2);$$

$$-3 + (+2);$$

$$3 - (-2);$$

$$-3 - (+2);$$

$$-3 + (+3);$$

6. Compare without calculation.

a.  $100 - (35 - 20)$      $100 - (35 + 20)$

b.  $100 + (35 - 20)$      $100 + (35 + 20)$

c.  $100 - (-35 - 20)$      $100 - (-35 + 20)$

d.  $100 + (-35 - 20)$      $100 + (-35 + 20)$

7. Positive or negative number will be the product of

- a) Two negative and one positive numbers.
- b) One negative and two positive numbers
- c) Three negative numbers.
- d) Three positive numbers.

8. 25 identical thick books or 45 identical thin books can fit on a bookshelf. Will there be enough space on a bookshelf for 20 thick and 9 thin books?

9. Numbers 100 and 90 are divided by the same number. In the case of 100, the remainder is 4, in the case of 90, the remainder is 18. What is the divisor?