MATH 6: HANDOUT 21
MATH BATTLE

1. The vertices of a triangle are $A(4,3)$, $B(6,-1)$, $C(-2,-5)$. L,M are midpoints of BC and CA. Find the coordinates of L and M and show that $LM = \frac{1}{2}BA$

2. (a) Draw the graph of the equation $x^2 + y^2 - 1 = 0$.
(b) Draw the graph of the equation $x^2 + (y-1)^2 - 1 = 0$.
(c) Draw the graph of the equation $(x+2)^2 + (y+3)^2 = 4$.
(d) Draw the graph of the equation $xy = 0$.

3. Prove:

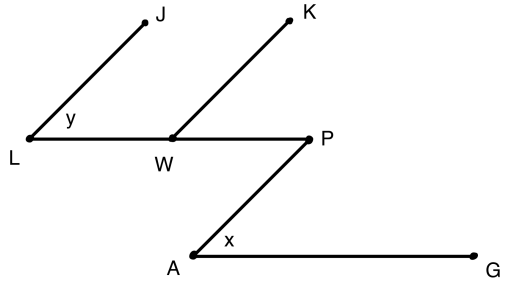
Theorem 20. *Let $ABCD$ be a quadrilateral such that opposite sides are equal: $AB = CD$, $AC = BD$. Then $ABCD$ is a parallelogram.*

4. Sketch the following function:

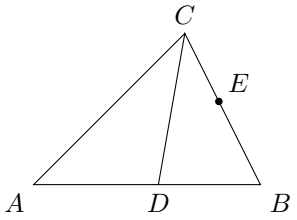
$$f(x) = \begin{cases} x^2 & \text{if} \quad x \leq 0 \\ x & \text{if} \quad x > 0 \end{cases}$$

5. Show that each rhombus is a parallelogram.

6. Assuming that $\overline{LJ} \parallel \overline{WK} \parallel \overline{AP}$ and that $\overline{PL} \parallel \overline{AG}$ in the following figure, prove that $m\angle 1 = m\angle 2$



7. Given $\triangle ABC$, let D be on AB such that $\angle ACD$ is equal to $\angle BCD$. Suppose we wish to place a point E on $\overline{BC}$ such that $\triangle CED$ is isosceles. Prove then that we must have $AC \parallel DE$.



8. Show that if, in a quadrilateral $ABCD$, diagonally opposite angles are equal ($m\angle A = m\angle C$, $m\angle B = m\angle D$), then opposite sides are parallel. [Hint: show first that $m\angle A + m\angle B = 180°$.]

9. In a trapezoid $ABCD$, with bases AD, BC, let E be the midpoint of side AB and F be the midpoint of side CD. Show that $EF \parallel AD$. [*Optional:* Can you prove that $EF = \frac{1}{2}(AB + CD)$? The figure on the right may help.]