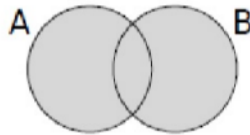# MATH 6
SETS II

We already learned that *sets* are collections of object. Objects in the sets are regarded as *elements* of the sets. It is possible to have operations between sets.
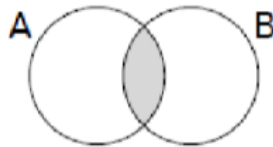
Operations between sets. $A \cup B$: union of A and B. It consists of all elements which are in either A or B (or both):

$$A \cup B = \{x \mid x \in A \text{ OR } x \in B\}.$$

$A \cap B$: intersection of A and B. It consists of all elements which are in both A and B:

$$A \cap B = \{x \mid x \in A \text{ AND } x \in B\}.$$

$\overline{A}$: complement of A, i.e. the set of all elements which are not in A: $\overline{A} = \{x \mid x \notin A\}$.
$|A|$: number of elements in a set A (if this set is finite).

In your last homework, we learned that

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Subsets. We say that set A is a subset of B (notation: $A \subset B$) if every element of A is also an element of B: $x \in A \Rightarrow x \in B$. Note that A can be equal to B.

HOMEWORK

1. Let $A = \{1, 2, 3\}$. Find all the subsets of A. Do not forget the empty set $\emptyset$ and set A itself.

2. Let
   A=set of all people who know French
   B=set of all people who know German
   C=set of all people who know Russian
   Describe in words the following sets:
   (a) $A \cap B$ (b)$A \cup (B \cap C)$ (c) $(A \cap B) \cup (A \cap C)$ (d) $C \cup \overline{A}$.

3. Let us take the usual deck of cards. As you know, there are 4 suits, hearts, diamonds, spades and clubs, 13 cards in each suit.

Denote:

H=set of all hearts cards

Q=set of all queens

R=set of all red cards

Describe by formulas (such as $H \cap Q$) the following sets:

all red queens

all black cards

all cards that are hearts or a queen

all cards other than red queens

How many cards are there in each set?

4. In a class of 25 students, 10 students know French, 5 students know Russian, and 12 know neither. How many students know both Russian and French?

*5. A local frog named Filo running for parliament tells the public, at the famous amphibian debate convention, that Filo will speak for all frogs who do not speak for themselves, and Filo will speak for no one else. Can you logically deduce if Filo will speak for Filo's own self?

6. Consider the following sets:

$\mathbb{Z}$ — all whole numbers (positive and negative)

$\mathbb{N}$ — all positive whole numbers

$\mathbb{R}$ — all numbers

$\mathbb{Q}$ — all rational numbers (i.e., those that can be written as a fraction)

Order them from smallest to largest, so that each set is a subset of the next one.

7. Find sets A, B, C if you now that $A \cup B = \{1, 3, 4, 5, 7\}$, $B \cup C = \{1, 2, 4, 5, 6, 8, 9\}$, $(A \cup B) \cap C = \emptyset$, $(B \cup C) \cap A = \{1, 5\}$.

8. Find A if you know that $A \cup \{5, 7\} = \{3, 5, 7, 8\}$, $A \cap \{1, 2, 5, 7\} = \{5, 7\}$